UCRL - 74814
PREPRINT

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

ELECTROMAGNETIC PULSE — NUMERICAL METHODS AND APPLICATIONS

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June 18, 1973

A Short Course on Numerical Techniques
For Antennas and Electromagnetics
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ABSTRACT

A brief overview of some numerical methods, primarily integral equation types which have been developed for or applied to nuclear EMP survivability/vulnerability assessment problems is presented. Results of sample calculations are presented for a variety of wire and surface structures in both the time and frequency domains. Also discussed are some of the problems associated with the development of an engineering approximation methodology for EMP applications.

FOREWORD

Preparation of the material presented here was supported by Defense Nuclear Agency under the auspices of the U.S. Atomic Energy Commission.

OUTLINE

EMP NUMERICAL APPLICATIONS

I. INTRODUCTION

Numerical methods such as those based upon various integral equation formulations have been employed for both basic and applied electromagnetic research with increasing emphasis in recent years. There are of course diverse reasons for this, including extended machine size and capability, the greater availability of the computer time itself, dissemination of information pertaining to such methods, etc. As a result, it is now rather common to find one or another version of the computer programs developed by early workers in the area to be employed on a regular basis for antenna analysis and design, coupling and interaction studies etc. It goes without saying that this increasing diversity and breadth of application would not follow had these computer oriented methods not been validated by extensive comparison of computational results with experimental data.

One of the more demanding areas where numerical methods can be of real use is that involved in the susceptability/vulnerability assessment of systems exposed to the electromagnetic pulse (EMP) from a nuclear explosion. Under certain conditions this pulse may result in plane wave field amplitudes, at great distances from the explosion itself, on the order of 50,000 volts per meter with rise times on the order of 10 nanoseconds. A nominal time domain plot of an EMP wave form and its corresponding frequency spectrum are shown in Figure 1 [Nelson (1971)].

Analytic expressions for the wave form and spectrum shown in Figure 1 are given by

$$E(t) = E_o\left[e^{-\alpha t} - e^{-\beta t} - A(e^{-\gamma t} - e^{-\delta t})\right]$$

where

t = time in seconds

$\alpha = 1.5 \times 10^6$

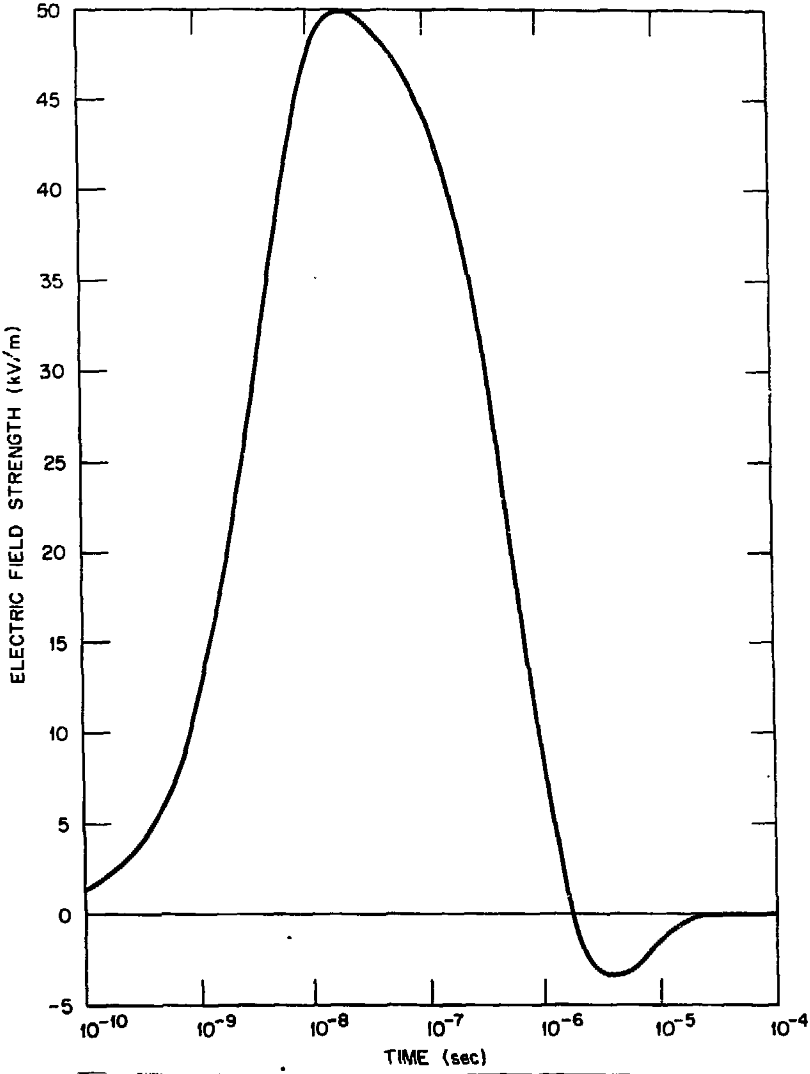
$\beta = 2.6 \times 10^8$

$\gamma = 2.0 \times 10^5$

$\delta = 5.0 \times 10^5$

$$A = \frac{\alpha^{-1} - \beta^{-1}}{\gamma^{-1} - \delta^{-1}}$$

E_o - $5 \times 10^4/0.9646$ volts/meter



The Representative EMP

FIGURE 1a. Waveform

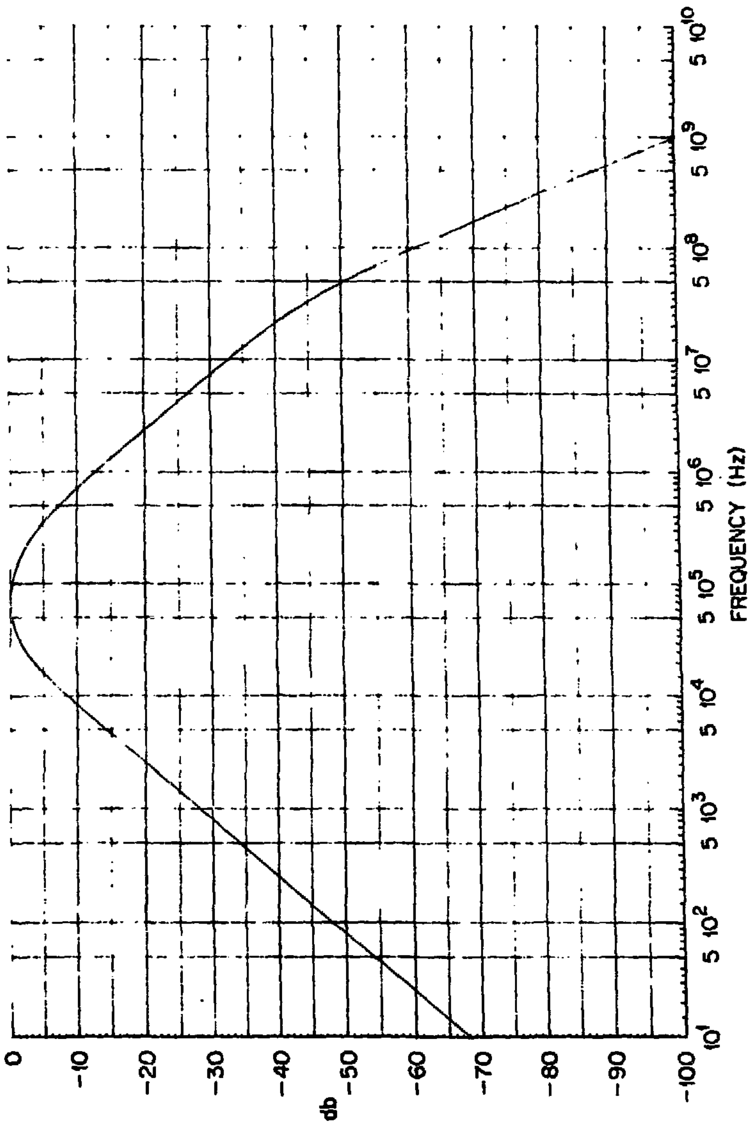


FIGURE 1b. Normalized power spectrum (after Nelson, 1971)

and

$$E(\omega) = \frac{E_0}{\sqrt{2\Pi}}\left[\frac{1}{\alpha + i\omega} - \frac{1}{\beta + i\omega} - \frac{A}{\gamma + i\omega} + \frac{A}{\delta + i\omega}\right]$$

respectively.

Systems which may be illuminated by an EMP pulse could include a missile site, a missile in flight, a communication facility, a naval ship, a computing facility, an aircraft in flight, etc. Each of these particular kinds of systems may require its own specific type of hardening to render the system insensitive to the EMP. In the following sections we discuss some of the kinds of problems which face an EMP analysist/designer and illustrate application of some numerical methods to representative problems.

II. PRELIMINARY CONSIDERATIONS

While numerical methods based primarily on integral equation formulation, such as those discussed here have been successfully applied to a variety of both wire and surface problems, as well as hybrid geometries, their practical use has for the most part involved only structures whose largest characteristic dimension is on the order of one or a few wave lengths. Furthermore these methods have for the most part been applied to what might be termed idealized or homogeneous structures which consist of perfectly conducting metal surfaces where the effects of imperfect conductivity can be represented in terms of impedance loads. Thus both the structure description required for computer treatment as well as the formulation and numerical techniques required to reduce the formalism to numerical results are, while computationally involved, relatively straightforward in terms of the structure's physical complexity.

In contrast to this, a typical EMP problem application may involve considerably more complicated structures both on a size and complexity basis and furthermore may extend over frequency ranges covering the Rayleigh region, resonance region and extending upwards well into the geometrical optics region for the EMP spectrum shown in Figure 1. As a specific example we may mention the survivability/vulnerability assessment of an aircraft such as the B1 bomber when exposed to an EMP waveform. The aircraft structure may have its fundamental resonance in the low VHF spectrum corresponding essentially to that of a dipole scatterer. However the frequency spectrum of concern extends well over a decade higher where the aircraft obviously then presents a much more demanding analysis problem to the EMP engineer. In this higher frequency region the EMP problem is not unlike that which has been pursued for many years from a radar scattering viewpoint where the cross section of an aircraft-like geometry might be required for frequencies for an aircraft many times the wavelength in size. But where the radar cross section analysis problem is essentially

complete once the surface induced sources or possibly only an estimation of the scattered fields at large distances from the target have been obtained, the EMP design engineer's job has just begun. For it is the effect of these induced sources over the surface of the structure which must be determined in the aircraft interior to quantitatively describe the threat which the incident EMP imposes.

It might be claimed then that the EMP engineering analysis requirements can be considerably more complicated, demanding and intractable than those encountered in the more familiar field of radar scattering. Yet the aircraft problem itself represents one of the simpler examples which can be extracted from the EMP category. Perhaps a more typical example problem faced by an EMP engineer is that of determining the survivability/vulnerability of a communications facility illuminated by the same EMP waveform. In this particular case a building whose characteristic dimensions may be 20 feet by 20 feet by 10 feet high with walls made of concrete to which rebar has been added for strengthening; whose interior components may be connected to the exterior via cables, pipes, etc.; and in whose walls are located various apertures associated with doors, ventulation ducts, etc., must all be considered in determining the interior threat posed by the EMP.

As yet no engineering-like simple approximations have been developed for general application to this latter problem. Various concentrated efforts have been made in connection with specific system requirements to solve problems of this kind. One example can be cited, the MINUTEMAN system. In this case an attempt has been made to analyze or model the various components in the system in a very deterministic fashion. Large circuit analysis programs are used to determine the currents flowing into the various component racks resulting from the EMP illuminating exterior communications antennas, sewer lines, missile silos, connecting tunnels, etc. That this approach can be considered at all is primarily due to the fact that there are many nearly identical installations involved in the MINUTEMAN system so that a model developed in such great detail may possibly be used for the analysis of all of these separate installations. There are unfortunately many areas where the differences between various installations in given systems may be considerably greater than their similarity. As a consequence the deterministic description and analysis which has been used for the MINUTEMAN problem may not be feasible in general.

Engineering-like approximations are required to develop tools for the analysis of the latter kind of problem. This is a task which is still being carried out and for which no really workable methodology has been developed. Attempts are being made to systematize an approximate description of such systems with accuracy targets of the order of 10 dB or possibly 20 dB, for the quick-and-dirty assessment of such installations. Eventually it is likely that such attempts will meet with some success and that the final approach to this particular class of problem will be based on a combination of statistics and probability theory together with extractions from electromagnetics, numerical

analysis, circuit codes, etc. In this regard significant contributions can be expected to be obtained from the types of deterministic numerical analysis under consideration here, for these numerical techniques permit what is essentially computer experimentation to derive the sensitivity of these sought-for solutions to various of the important problem parameters. Thus while the techniques we are discussing may not be used to solve EMP problems in total they should serve in a key way to permit the eventual realization of a general EMP methodology.

It is thus relevant to consider numerical techniques within the context of the overall EMP problem requirements. We will do this in the following section where examples are presented of numerical applications to various EMP problems suited to the relatively limited capabilities current numerical techniques provide. In considering the selection of the following examples emphasis has been placed on obtaining numerical results for application to what appear to be relevant practical problems. We furthermore restrict our attention to applications which involve answering the EMP survivability/vulnerability question as opposed, for example, to numerical applications to EMP simulator design. For extensive presentations concerning both the area of simulator design as well as the vast body of special theoretical developments which have been worked out within the framework of EMP, the reader is referred to the series of AFWL notes in the various EMP topic areas.

III. APPLICATIONS

Integral equations and other numerically oriented methods have thus far been quite widely applied to a number of military programs which require an EMP survivability/vulnerability assessment. Included among the programs which have exploited such numerical techniques are the MINUTEMAN System, Polaris and Poseidon Missile Systems, Nike X and Safeguard Systems, the B1 Bomber, 747 AWACS and other aeronautical systems, the Pershing Missile, Office of Civil Defense Emergency Operating Centers, etc. Obviously, information concerning the numerical analysis of such problem areas where specific system's performance is involved cannot be discussed here. However, it is possible to cite general methods of approach and to present some representative numerical results for less systems specific, and thus less sensitive, operational aspects. The results included here cannot then be viewed as providing a detailed cross sectional view of current EMP survivability/vulnerability assessment practices but instead provide an indication of the problems being considered and represent, so to speak, the tip of the EMP iceburg. It should also be kept in mind that even though many aspects of these problems are not classified, access to that information and its subsequent dissemination requires the permission of the various project offices involved, which may not be readily attainable for presentations such as this.

Before getting into the specific numerical examples that we will eventually discuss, it is worthwhile to consider some of the broader aspects of

performing an EMP survivability/vulnerability assessment. Obviously, the problem requirements and hence the complexity of the associated analytical approach are very much determined by the details of the system under study. At one end of the range of difficulty we might consider the EMP response of an isolated VHF communications antenna when illuminated by an EMP and the resultant threat which its connected communications equipment will be exposed to. This particular problem may be solved in terms of the integral equations methods considered here alone, with possibly no further analysis required to determine the threat level or the hardening requirements of the associated communications gear. Some additional degree of modeling realism may of course be introduced by employing one of the now standard circuit codes, such as Sceptor, to determine the response of the input circuitry of the receiver or transmitter when analyzed in connection with the transient response of the antenna itself.

At the other end of our scale of problem difficulty, we might consider the example of the MINUTEMAN System. In this instance the scope of the problem has vastly increased to include the silos, connecting tunnels, a myriad of electronic gear, communications antennas, power lines, and the missiles themselves — all of which may be spread over an area of several square miles. We cannot consider this particular problem in any detail here of course, but it may be illuminating to mention the major steps used in analyzing this particular system by the analysts involved. The problem, understandably, has been broken up into a number of separate blocks, each of which itself may require the use of complicated computer programs for its analysis. The first block may involve determing the response of entry ports for EMP energy associated with the power lines and various communications antennas and, perhaps, the tunnels and sewer lines. The second block in this system may include the effects of the various non-linear protection devices such as electric surge arrestors, or ESA's, when exposed to the input wave forms determined from step 1. Block 3 which follows, could be a deterministic circuit description of the cables and interconnecting wires which form paths over which the EMP energy may reach identified critical circuitry, with the analysis performed via various circuit codes. The scope of the problem at this particular juncture has increased to a considerable magnitude with the matrices resulting from the use of this deterministic model involving perhaps several thousand nodes in the equivalent circuits. This kind of detailed description of the interconnecting circuitry permits determining the voltage and current characteristics at any desired point within this network structure. As an alternative, one might consider simplifing the problem by characterizing the network via the transfer function between the input and output with the details of the intervening circuitry ignored. An advantage of the former approach is that it is possible to identify areas within this circuitry where protection devices may be installed at potentially great benefit in the overall hardening scheme. Finally, a fourth block, in which the various already identified critical circuit components have been selected for modeling again in a non-linear sense, concludes the model. It is at this stage that the threat to these sensitive mission critical components can be determined and hardening procedures identified.

Although very sketchy, this describes the basic approach used for analyzing the MINUTEMAN System in terms of its EMP response. The actual details of the final analytical procedures may differ from what has just been described, but the overall intent of the approach, which is to provide a highly detailed numerical model of the system, is essentially as given above.

For the remainder of our discussion concerning applications of numerical methods to the EMP, we will restrict our attention to a much less ambitious scope of problems. We will, in particular, be primarily concerned with what might be termed transfer admittance determination; that is, determining the current excited at a particular point or points on a structure due to illumination by an incident EMP. This step may of course represent but a first one in a sequence of operations involved in determining the threat, that is, the voltage or current, to which some particular circuit component may be exposed. In the case of the aeronautical problem for example, the transfer admittance provides us with only the surface currents and charges induced on the aircraft structure. Subsequent operations are required to obtain the interior fields, voltages, and currents which result from these surface induced sources and which eventually reach the various black boxes within the system.

In connection with the energy coupled to the structure interior, we might identify three primary coupling modes or points of entry. These are: 1) diffusion through the structure walls (which are generally intended to provide shielding); 2) aperture coupling through either actual holes in the walls or imperfections due to such things as welds, or imperfectly conducting contacts around the finger stock of a door; and 3) penetrating conductors, which may be associated with external power lines, communications antennas, and the like, which of necessity must penetrate the exterior shield and therefore provide the possibility for current injection to the interior. Each of these phenemena is itself a complicated problem, perhaps exceeding the difficulty of that induced in finding the surface sources. At this point therefore, we decrease the scope of our consideration still further by concerning ourselves primarily with the external coupling or transfer admittance problem. After discussing this topic in some detail, we will briefly consider the three energy injection mechanisms mentioned above. We will then conclude with a short discussion of the threat determination problem in terms of the interior energy distribution.

A. Transfer Admittance

1. Wire Structures in Homogeneous Media

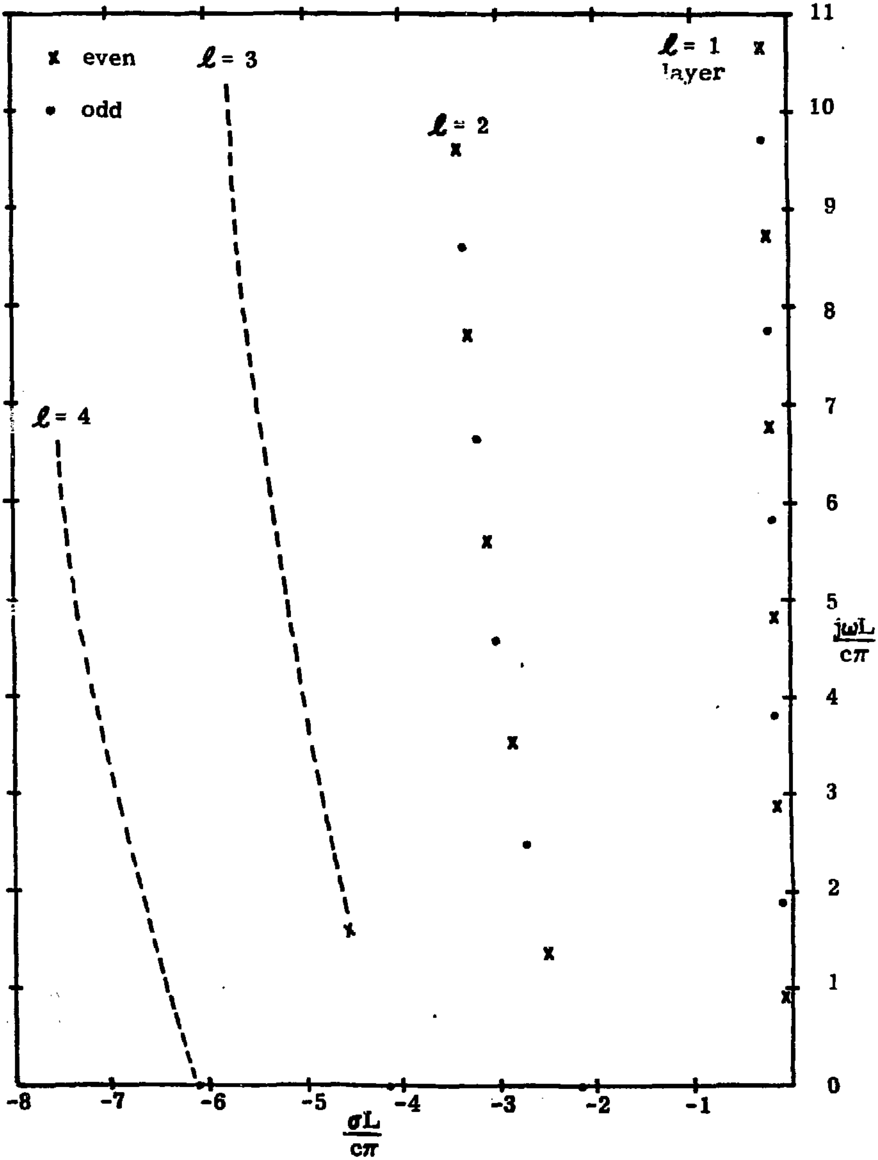
Quite understandably, perhaps the first geometry treated in an EMP context is that of the linear dipole. This particular antenna together with its closely related magnetic counterpart, the circular loop, has been studied using a variety of approaches. Initial efforts concerning these particular antennas were aimed at deriving effective heights and impedance variations with

frequency. Such data, when presented in appropriate tabular, nomagraphic, etc., form, permits a ready determination of the short circuit currents or open circuit voltages which will be excited on such antennas by an incident EMP. These structures, furthermore, had the advantage of having been studied quite thoroughly, there being available rather simple closed form expressions for these pertinent quantities over the frequency and size ranges of interest.

While subsequent attention has, in the meantime, been diverted to more complicated antenna and structure types (aircraft models, for example) there has again been a renewed interest in the transient characteristics of the linear dipole antenna. A principle reason for this is that the linear dipole serves quite well as a problem for testing the viability of the singularity expansion method (SEM) as an alternative procedure for obtaining the transient characteristics of more general structures. This particular method, which is described in some detail by Teshe (1972a 1973), basically involves finding the complex resonance frequencies for the structure under consideration. The response of the structure to arbitrary time varying excitation can then in principle be built up in terms of a residue series which involves the pole locations in the complex frequency plane, and the corresponding coupling vectors.

An example of using the SEM method for determining the transfer admittance of a straight wire when illuminated at broadside incidence by a step field is shown in Figure 2. Part a of this figure depicts the pole locations in the complex frequency plane of a thin wire, and part b the time response of the current at the center of the wire to step function excitation with the number of poles used in deriving the response a parameter. The potential advantage of using the SEM method is the short-hand description it permits for determining the transient response of structures, whose poles have been obtained, when illuminated by arbitrary wave forms. For comparison, the time response obtained from a Fourier transform of the frequency domain solution is included in Figure 2c.

The linear scatterer is, while one of the simplest problems which may be treated using the numerical methods under consideration here still rather broadly useful in a variety of EMP problem applications. Determining the response of EMP antennas, deliberate or accidental, is, of course, one obvious example of its application. Another is furnished by the geometry of a missile in flight, whose low order resonances are predominatly those of a thin wire structure. By properly modeling the plume which is produced by the missile, it is also feasible to evaluate the influence of the missile's



Application of the Singularity Expansion Method to a Thin-Wire of d (diameter)/ L (length) = 0.01:

FIGURE 2a. pole locations in the complex frequency plane

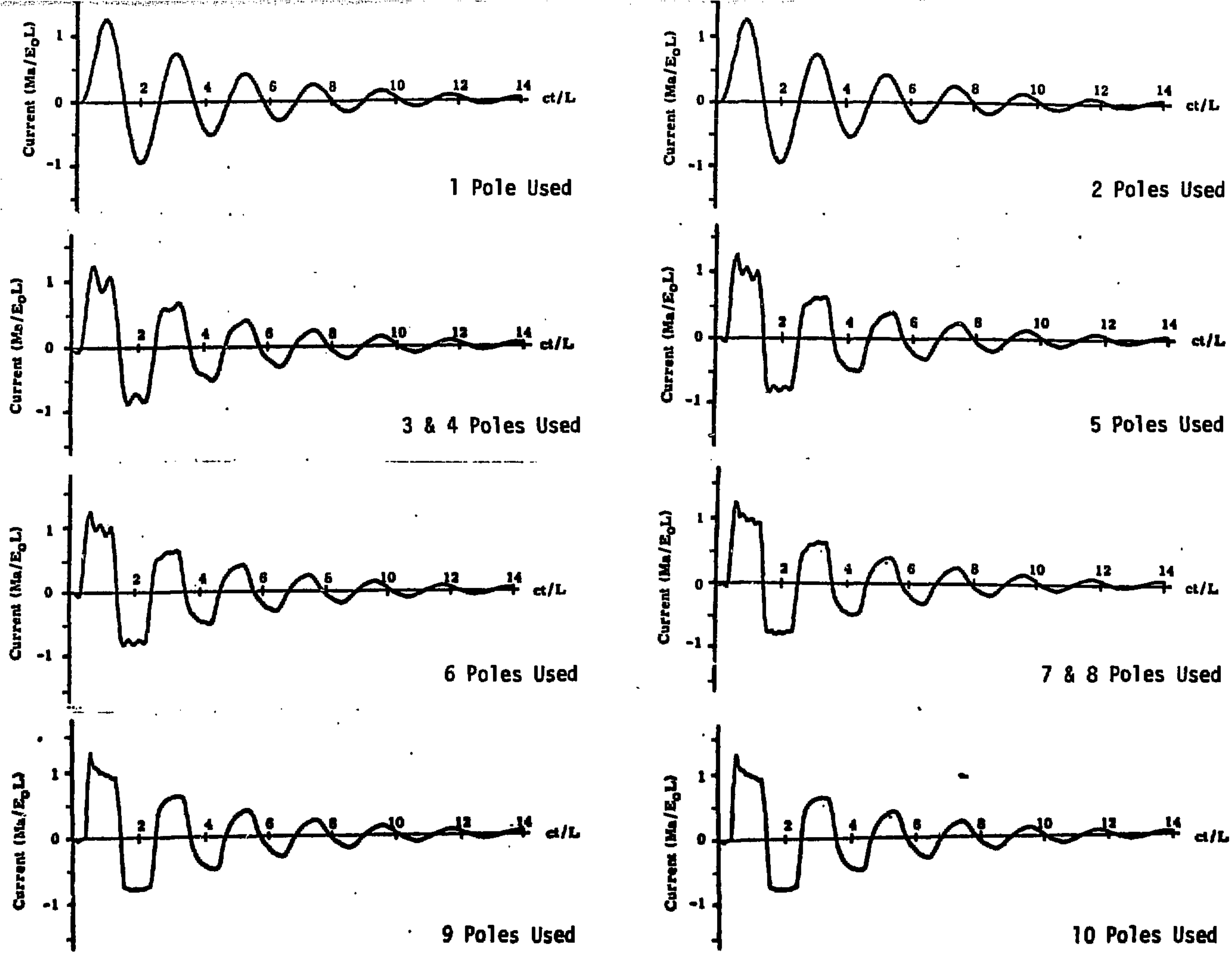


FIGURE 2b. Plots of the time response of the current at Z/L = 0.5 for plane wave incidence 30° from broadside with the number of poles a parameter.

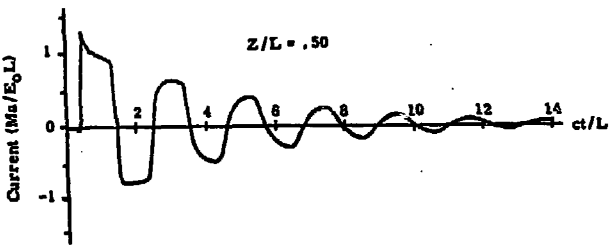


FIGURE 2c. Time response of the current obtained from a frequency-dependent integral equation and Fourier inversion for comparison with (b). (After Tesche, 1972a, 1973)

greater effective length when under powered flight and illuminated by an EMP, using an impedance-loaded thin wire as a model. An example of such an analysis is given by Harrison and Aronson (1969).

A still more demanding problem for EMP analysis concerns the response of, for example, a missile when exposed to the inner region of a nuclear burst where ionization currents may occur, the so-called ionization region. In this case, the EMP interaction phenomena become much more complicated because the medium itself is nonlinear. Merrewether (1971) has described a method developed for application to this particular problem, and demonstrates its application by considering the transient currents induced on a body of revolution in free space. His procedure, contrary to our predominant attention to integral equations, is based instead on a differential equation description using a finite difference procedure and an expanding mesh volume as the induced currents produce fields propagating outward from the illuminated cylinder. The general body shape considered by Merrewether is shown in Figure 3a, where it is assumed the thin wire approximation holds. Some representative numerical results he has obtained, for a unit step plane wave at broadside incidence are shown in Figure 3b. While Merrewether's approach is not as efficient as the more familiar integral equation formulation in either the time domain or frequency domain, for the free space case, it offers the only workable attack on the nonlinear medium problem, for which the conventional integral equation treatments are not suited.

In addition to exploiting related numerical developments in electromagnetics for the particular problem requirements of EMP, independent approaches have also been developed specifically oriented to EMP applications. Attention in these studies has been primarily directed to generalizing the Hallèn or vector potentional integral equation for the treatment of arbitrary wire structures. An account of some of this work is provided by Taylor, Lin and McAdams (1970), Taylor and Crow (1971), and by Crow, Shumpert and Taylor (1972), and Butler (1972). Representative numerical results from these studies are included in Figure 4.

The wire geometry treated by Taylor and Crow (1971) is shown in Figure 4a with the plane wave incident normal to the plane of two wires which are crossed at 90 degrees, and the direction of the electric field for the E and H polarizations as shown. The current on wire 2 as a function of $k\ell$ for the E-polarization case is shown in Figure 4b, and for the H-polarization case in Figure 4c.

An extension of the above treatment to handle non-perpendicular intersecting wires is reported by Crow, Shumpert and Taylor (1972).

Results from the finite difference technique for a body of revolution

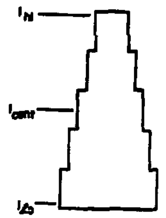


FIGURE 3a. Approximate body shape

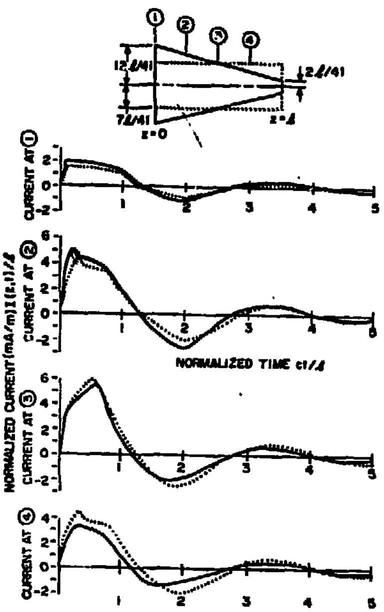
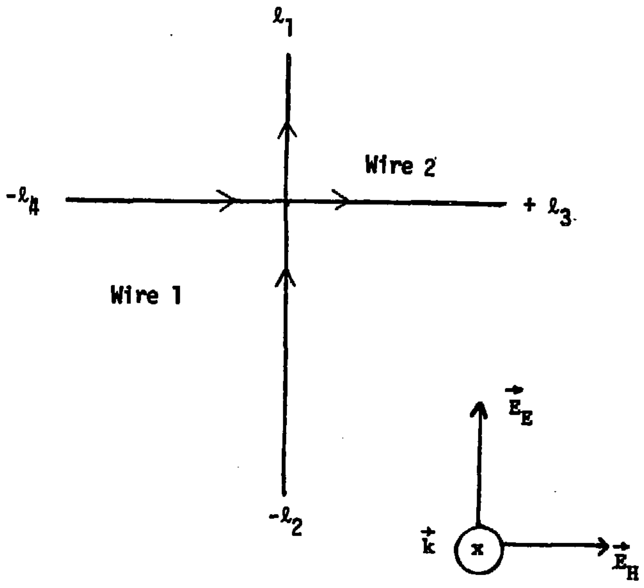


FIGURE 3b. The unit step response of a cone and cylinder. (After Merrewether, 1971.)



a = wire radius

$$\frac{a}{\ell_2} = 10^{-1}$$

$$\frac{\ell_1}{\ell_2} = 0.5; \quad \frac{\ell_3}{\ell_4} = 1.0$$

$$\frac{\ell_3}{\ell_1 + \ell_2} = 0.5$$

Scattering from perpendicular crossed wires for normal incidence:
FIGURE 4a. Problem geometry

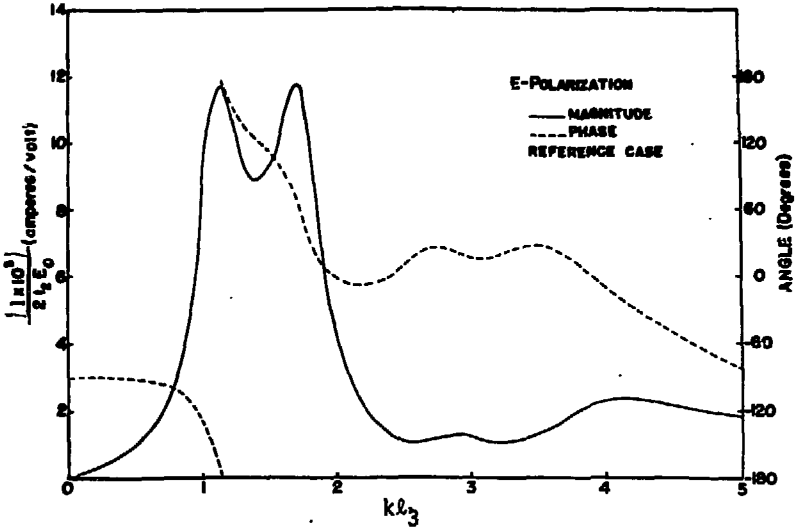


FIGURE 4b. Junction current on wire 2 vs $k\ell_3$ for E-polarization

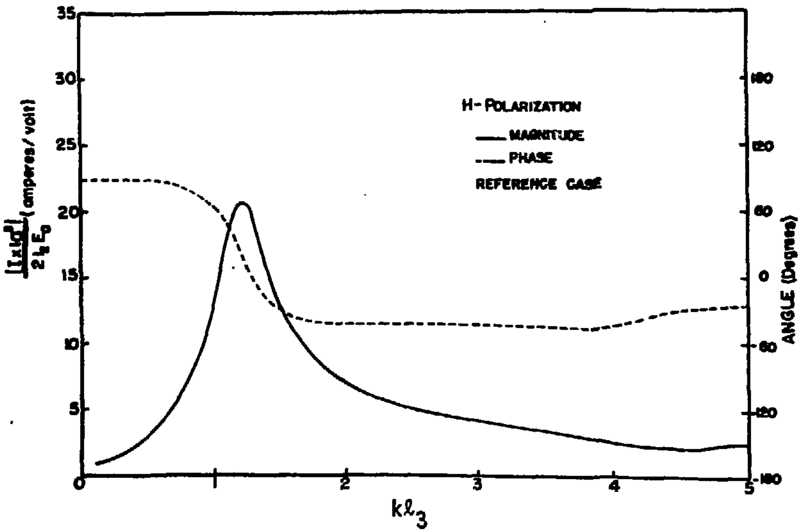


FIGURE 4c. Junction current on wire 2 vs $k\ell_3$ for H-polarization (after Taylor and Crow, 1971).

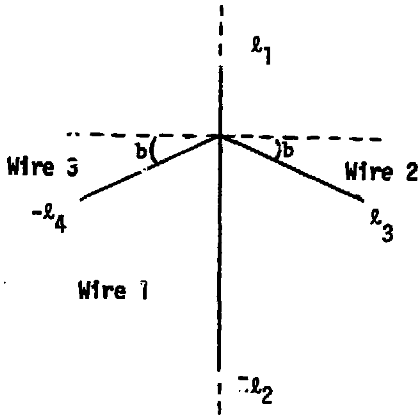
The geometry they considered is shown in Figure 5a. Results obtained from normal incidence of a plane wave to the plane of the wires and for the E- and H-polarizations are shown in Figures 5b and 5c respectively. Also included in Figure 5b is the reference case for perpendicular crossed wires. The consequence of altering the intersection angle b (relative to perpendicular) of the wires is small for b = 5 degrees, but is fairly pronounced at b = 30 degrees.

Note that these particular examples deal with wire structures having junctions of two or more wires, a feature certainly essential if these procedures are to be applied to aircraft modeling.

These perpendicular cross wire results were later challenged by Chao and Strait of Syracuse who, about the same time, generallized the original program due to Harrington to permit treatment of structures having multiple wire junctures as well, but using a very different procedure from that employed by the above authors. Although the initial numerical results presented by these authors were in disagreement, the discrepancies have been resolved as reported by Chao, Strait and Taylor (1971). Some current distributions given by Chao and Strait are included in Figure 6, again for normal wave incidence and with the electric field in the vertical direction.

Additional discussion concerning the various forms of the wire integral equations and its application to more general wire structures are given by Crow and Schumpert (1972) and Schumpert, Crow, and Taylor (1972), where multiple junction configurations in free space are further considered. The former is concerned with a wire configuration having an arbitrary junction geometry. The latter deals with wires whose thickness may invalidate the usual thin-wire apporximation. Some results obtained by Schumpert, Crow and Taylor (1972) for the current on perpendicular crossed wires of varying thickness and illuminated by a normally incident plane wave are shown in Figure 7. Note the oscillation in the current near the wire end as the radius becomes larger. By introducing an end connection to accomodate the thicker wire, this oscillation is considerably reduced.

Another kind of antenna which thus far has not evidently received much attention concerning its EMP responses is the conical spiral. Since this particular antenna is used for a variety of applications its transfer admittance over the EMP band is of value in connection with EMP studies. Some representative results obtained by Landt and Miller (1973) using a time dependent integral equation formulation are presented in Figure 8. Here we find that the low frequency, out of band, response of this particular antenna is not insignificant,



Scattering from non-perpendicular intersecting wires for normal incidence:

FIGURE 5a. Problem geometry.
Same values as Figure 4a.

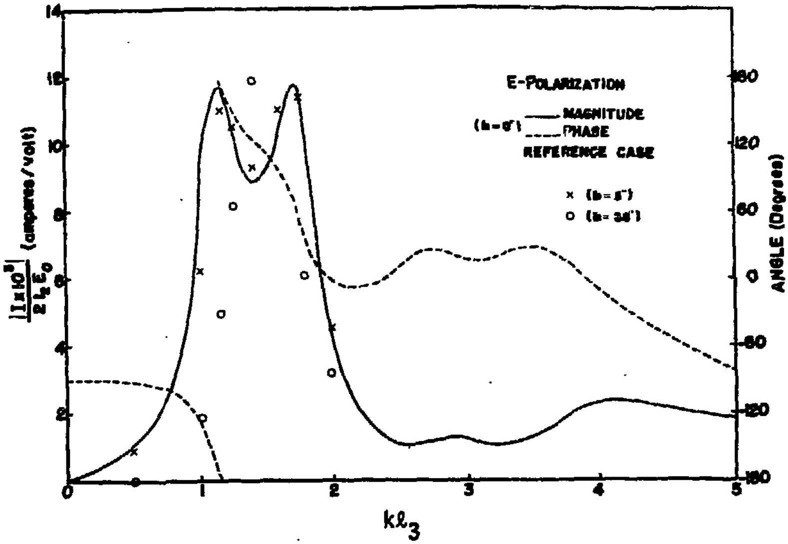


FIGURE 5b. Junction current on wire 2 vs $k\ell_3$ for E-polarization

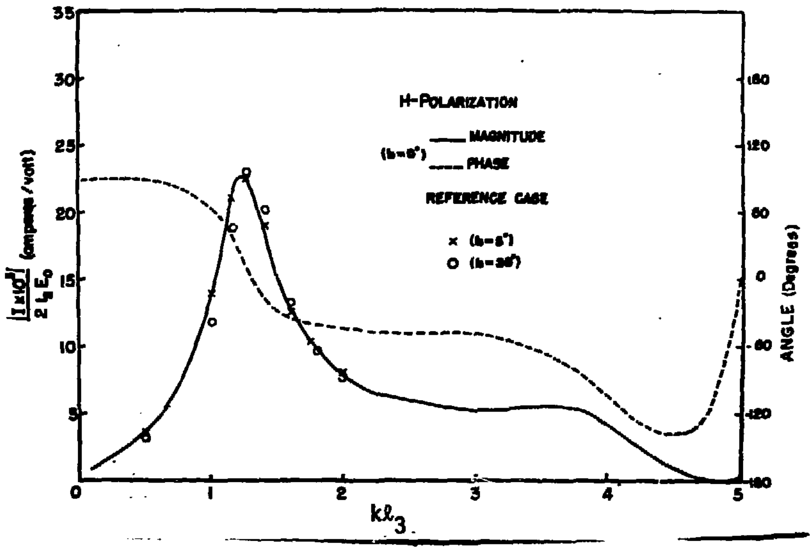


FIGURE 5c. Junction current on wire 2 vs $k\ell_3$ for H-polarization [see Figure 4a for field reference directions] (after Crow, Shumpert, and Taylor, 1972).

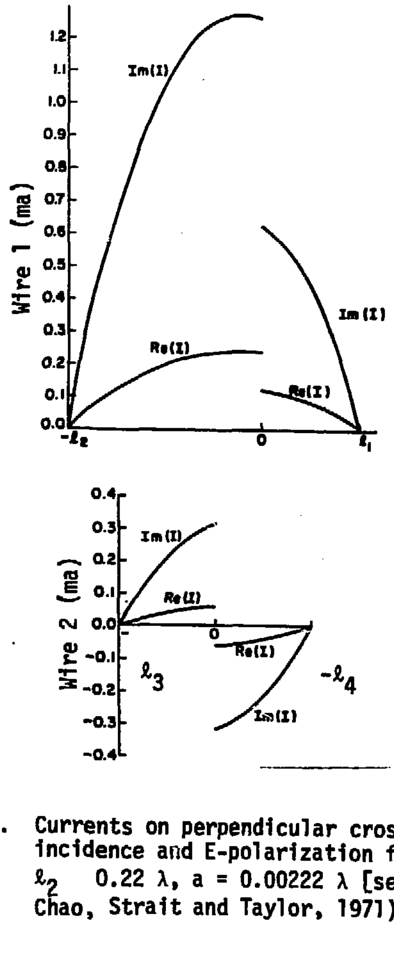
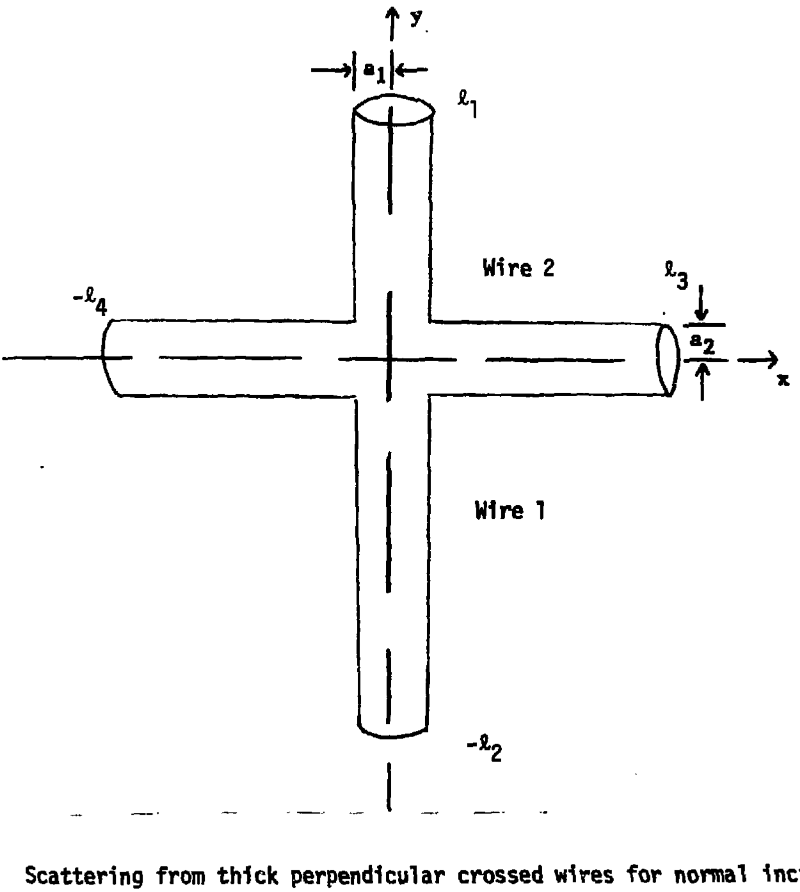


FIGURE 6. Currents on perpendicular crossed wires for normal incidence and E-polarization for $\ell_1 = \ell_3 = \ell_4 = 0.11\ \lambda$, ℓ_2 0.22 λ, a = 0.00222 λ [see Figure 4a] (after Chao, Strait and Taylor, 1971).



Scattering from thick perpendicular crossed wires for normal incidence

FIGURE 7a. Problem geometry

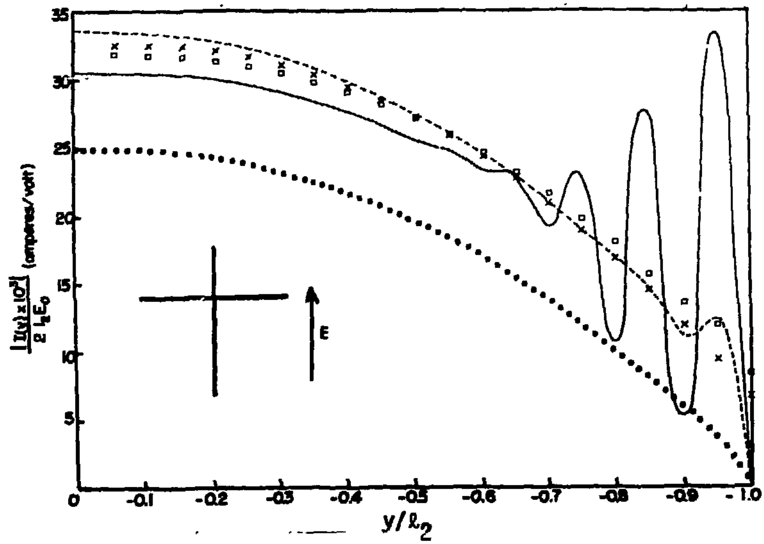


FIGURE 7b. Current on wire 1 for the E-polarization

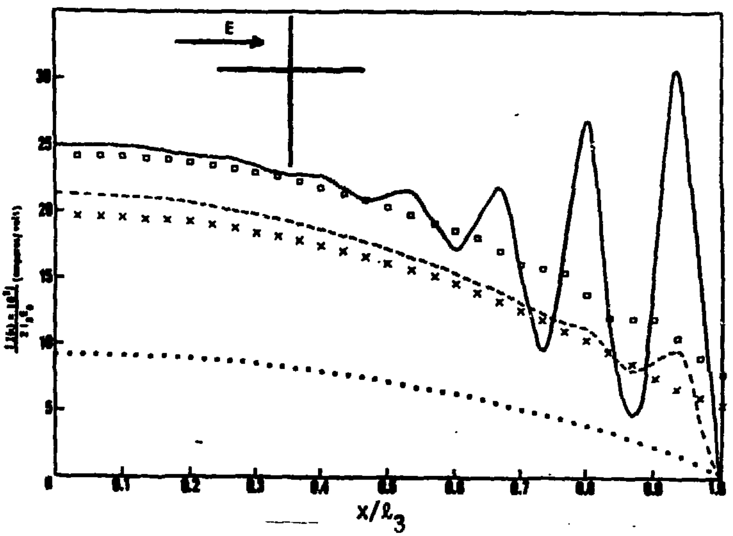


FIGURE 7c. Current on wire 2 for the H-polarization, for $k\ell_3 = 1.15$, $a_1 = a_2 = a$, the other parameters as in Figure 4a and ℓ_3/a = : 5 ——— (end correction □□ □); 10 — — — (end correction XXX); 50 · · · · · (after Schumpert, Crow, and Taylor, 1972).

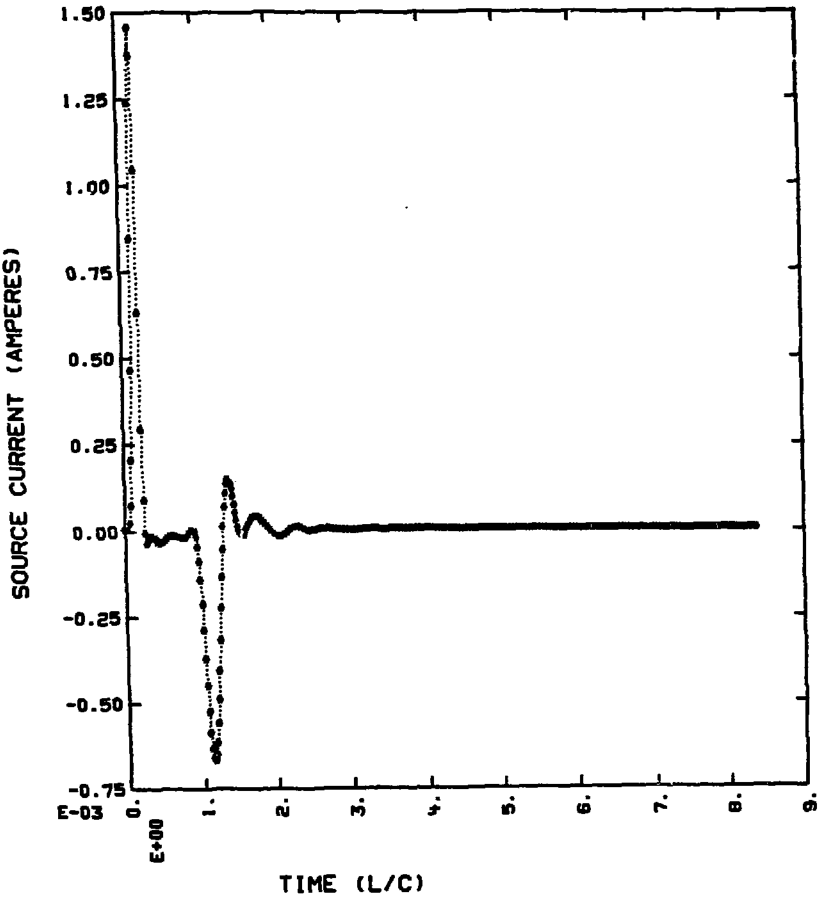
indicating that even conical spiral antennas whose operating bands are well above the predominant part of the EMP spectrum, might experience substantial energy collection and resultant circuitry response when exposed to an EMP field.

2. Wire Structure Near a Ground Plane

It is recognized that while wires in free space can serve as models for a surprising variety of practically interesting EMP problems, there are many instances where additional degrees of complexity must be considered. One of the most obvious extensions, of course, concerns the transfer admittance of a wire located near the ground plane. This problem is of interest for at least two reasons: 1) wires located near ground planes may be used as models for power lines, buried metal pipes and even tunnels small relative to the wave length. 2) the ground plane-wire interaction problem may be considered the first step in analyzing more complicated geometries, such as for example, an airplane parked on the runway and whose structure, at low frequencies, may be represented as an interconnection of straight, thin wires. The general treatment of this problem has been well documented by Sunde (1948). Application of his developments to various aspects of the EMP problem has been described by numerous authors, including Harrison and Houston (1969), Harrison (1969), Bechtold and Kozakoff (1970) and Harrison (1970). Experimental confirmation of the low frequency currents excited on wires lying on the ground as predicted by a transmission line theory has been given by Whitson and Vance (1965).

Various kinds of wire communications antennas may also present challenging modeling problems for EMP survivability/vulnerability assessment in connection with the ground proximity problem. An example, of this kind of problem is described by Deadrick, Landt and Miller (1973). Some numerical results from their analysis are included in Figure 9. Part a of the figure depicts the geometry of the fan doublet antenna, so named for obvious reasons, with the antenna shown relative to its height above the ground plane under normal operating conditions. Experimental data concerning the response of this antenna when illuminated by the field of a bi-cone simulator at Harry-Diamond Laboratories, Woodbridge Research Facility, were provided for comparison with the computed results by Stark and Klebers (1972).

Representative numerical results obtained from the modeling of this antenna via the electric field integral euqation and the three term current expansion [Gee et al. (1971)] are included in parts b, c, d, and e of Figure 9. Parts b and c are the transfer admittance of this particular antenna when located in free space. In part d is shown the feed segment current for excitation by an incident



Time domain derived results for a 2.5 turn, 10 degree half angle, conical spiral of 3 m total wire length.

FIGURE 8a. Approximate impulse response of source region current when excited as a transmitting antenna by the Gaussian pulse (0.1 level width ~ 1.67 nsec) of a 350 ohm generator.

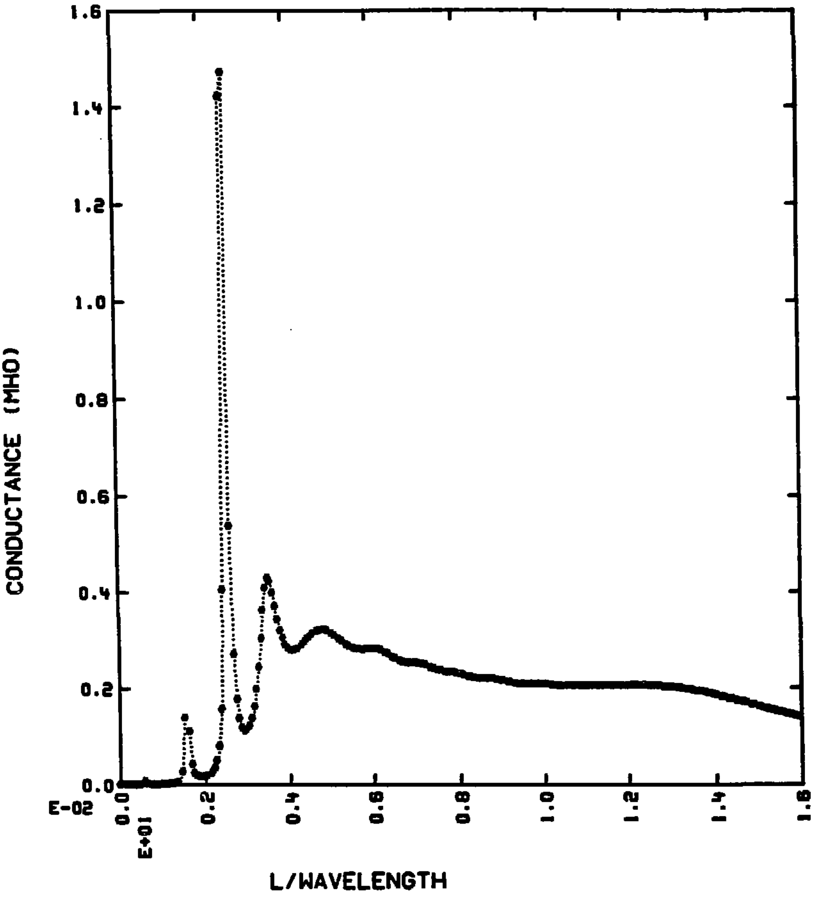


FIGURE 8b. Input conductance obtained from the Fourier transform of (a).

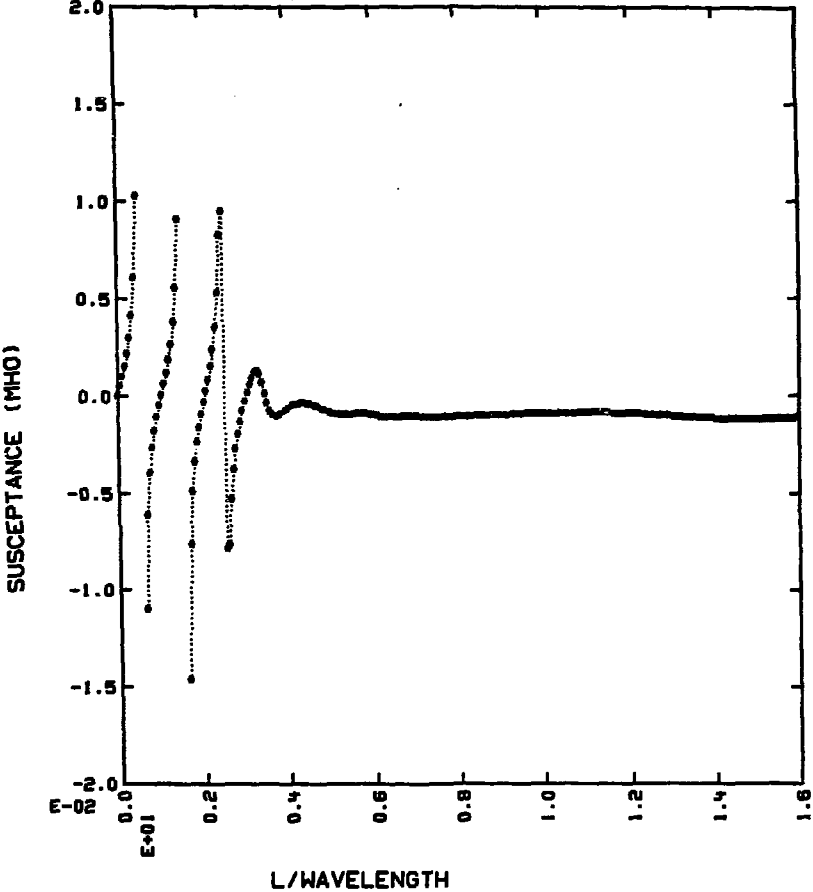


FIGURE 8c. Input susceptance obtained from the Fourier transform of (a).

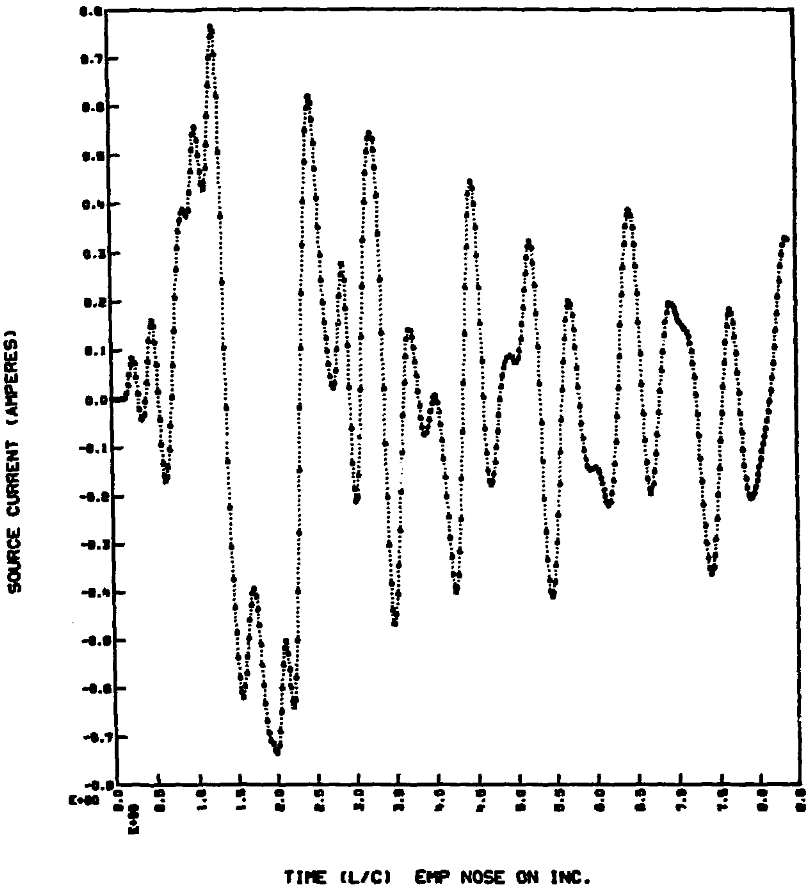


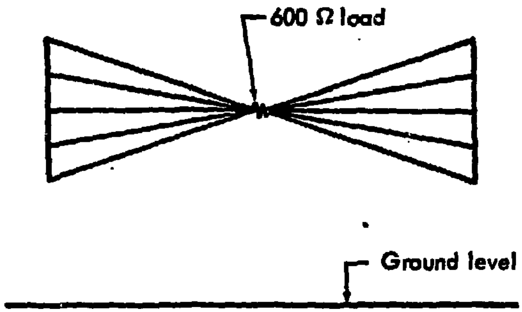
FIGURE 8d. Time response of the source region current for an axially incident EMP simulator (biconic) waveform having a peak amplitude of 5,600 V/m (after Landt, Deadrick and Miller).

plane wave having the time variation corresponding to that actually radiated by the bi-cone simulator at the antenna location. The numerical results in this case do not include the effect of the ground plane. In part d of the figure is shown the corresponding response of the fan doublet antenna illuminated by the same wave form, but including the ground interaction of the antenna in the transfer admittance calculations up to a frequency of 20 Megahertz. There may be seen a clear difference between part d and e, for times greater than 100 nanoseconds or so, corresponding to the propagation time for the fields excited by the antenna to propagate down to the ground and be reflected back to the antenna. For convience these two numerically derived curves are plotted together in Figure 9f. The accuracy of the particular model used here might be further improved upon by including the non-planarity and actual polarization of the bi-cone field over the entire fan doublet since its close proximity to the bi-cone simulator (approximately 300 feet) indicates that the field cannot be considered to be of plane wave nature and perpendicular incidence over the entire antenna structure.

It is in many cases adequate to consider the ground plane to be perfectly conducting, as a consequence of which a rigorously correct solution can be much more readily and efficiently obtained. The addition furthermore, of a second image plane parallel to the first permits the modeling of structures located in simulators in a fairly realistic way. Examples of the former application are given by Tesche (1972b) and Taylor (1972), while problems in the latter area have been considered Taylor (1967) Tesche (1971), and Scheer and Neureuther (1972).

Some results due to Tesche (1972) for a plane wave incident on a straight wire located over perfect ground are shown in Figure 10. The geometry of the problem is given in Figure 10a. A plot of the transfer admittance as a function of frequency is presented in Figure 10b and the corresponding time response for unit step excitation follows in Figure 10c, for the current at the center of the wire and the wire far removed from the ground plane. Note that the wire is alligned with the incident electric field and perpendicular to the ground plane when $\phi = 0$ and $\theta = 90$ degrees. The effect of moving the wire close to the ground plane is demonstrated by the results of Figures 10d and 10e where an h/L value of 0.5 is used.

An L-shaped wire over a perfect ground was studied by Taylor (1972). The problem's geometry is presented in Figure 11a, and the frequency variation of the transfer admittance is shown in Figure 11b for plane wave incidence and a vertical electric field (relative to the ground). The response of the L-wire appears to differ significantly from that for the straight wire structure.



Results computed from a frequency domain integral equation for the fan doublet antenna.

FIGURE 9a. Antenna geometry.

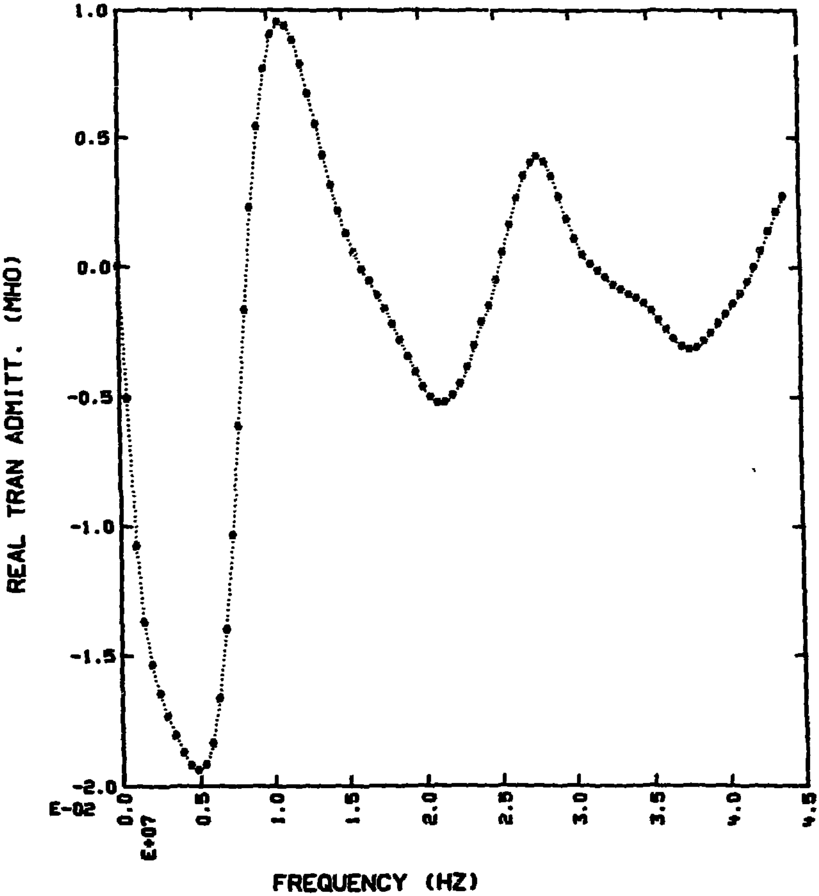


FIGURE 9b. Real part of the transfer admittance for broadside illumination of antenna in free space.

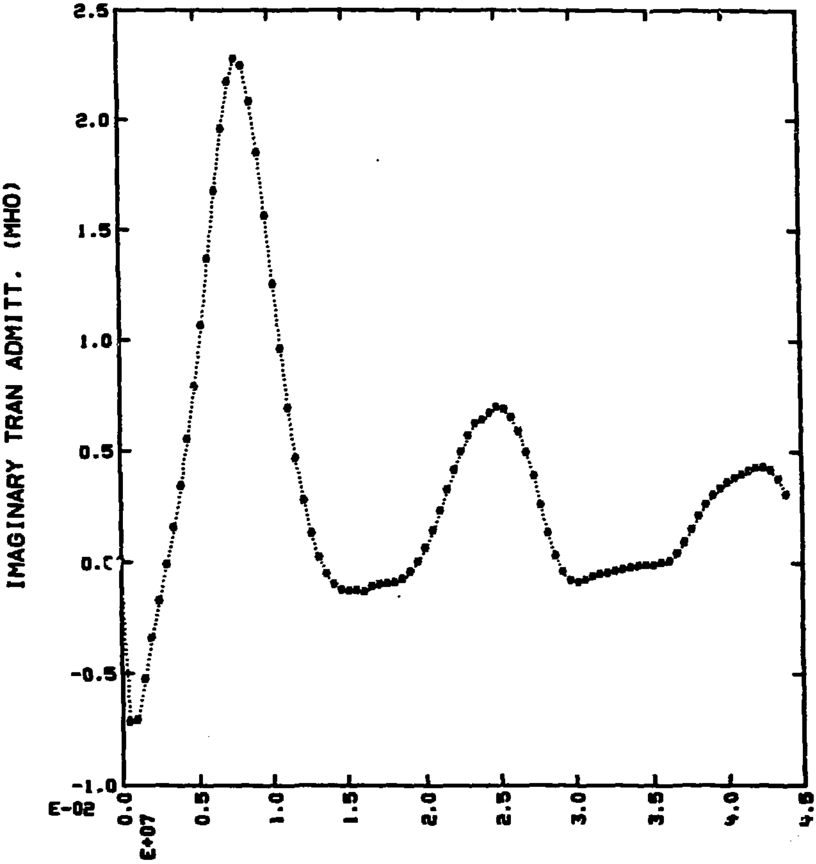


FIGURE 9c. Imaginary part of the transfer admittance for broadside illumination of antenna in free space.

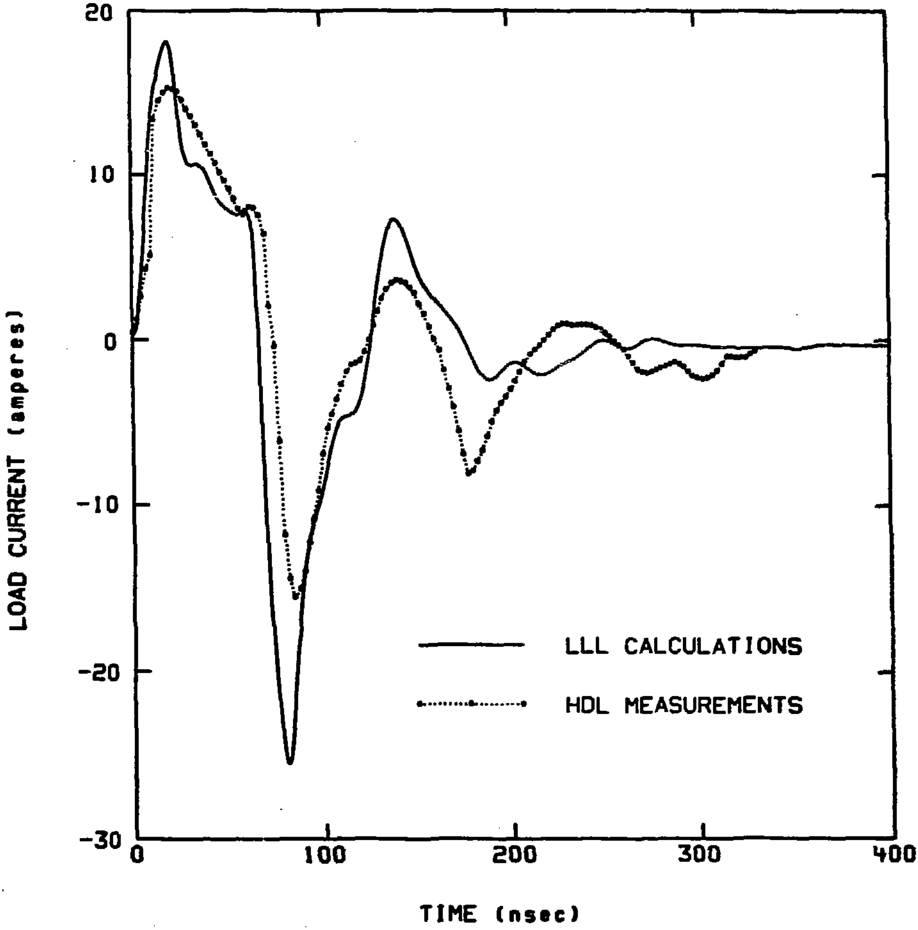


FIGURE 9d. Time response derived from free space transfer admittance and actual simulator wave form.

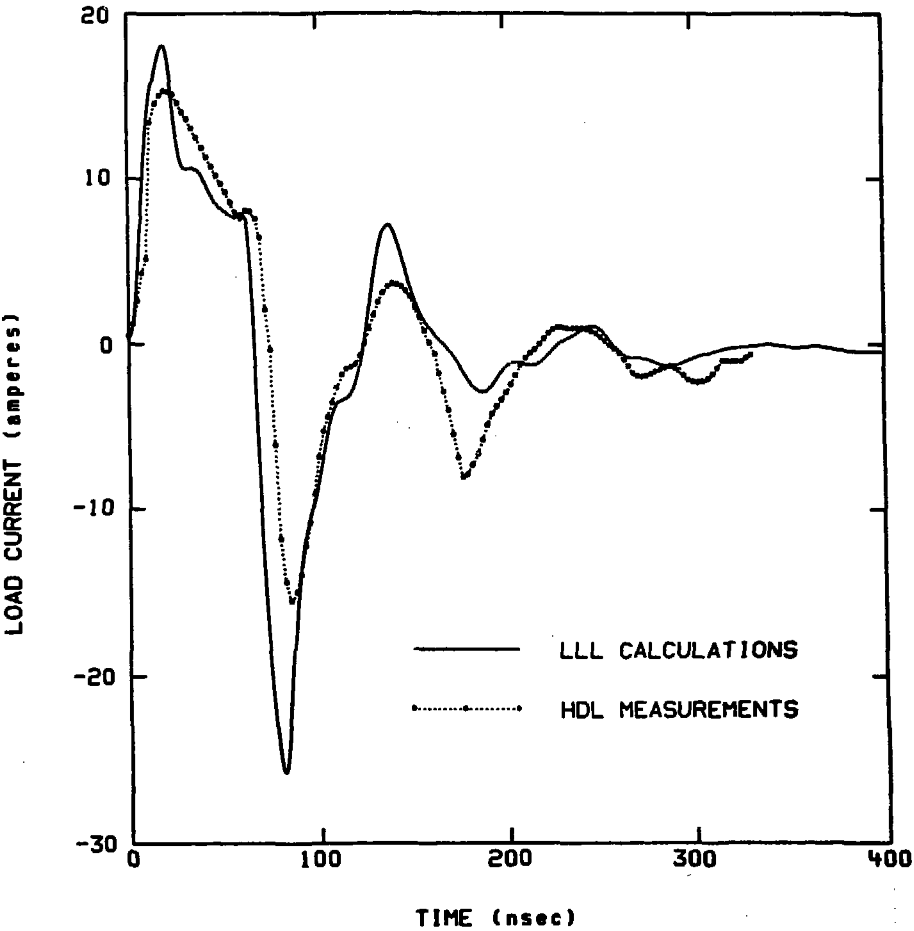


FIGURE 9e. Time response as in (d) but including ground effect in the transfer admittance.

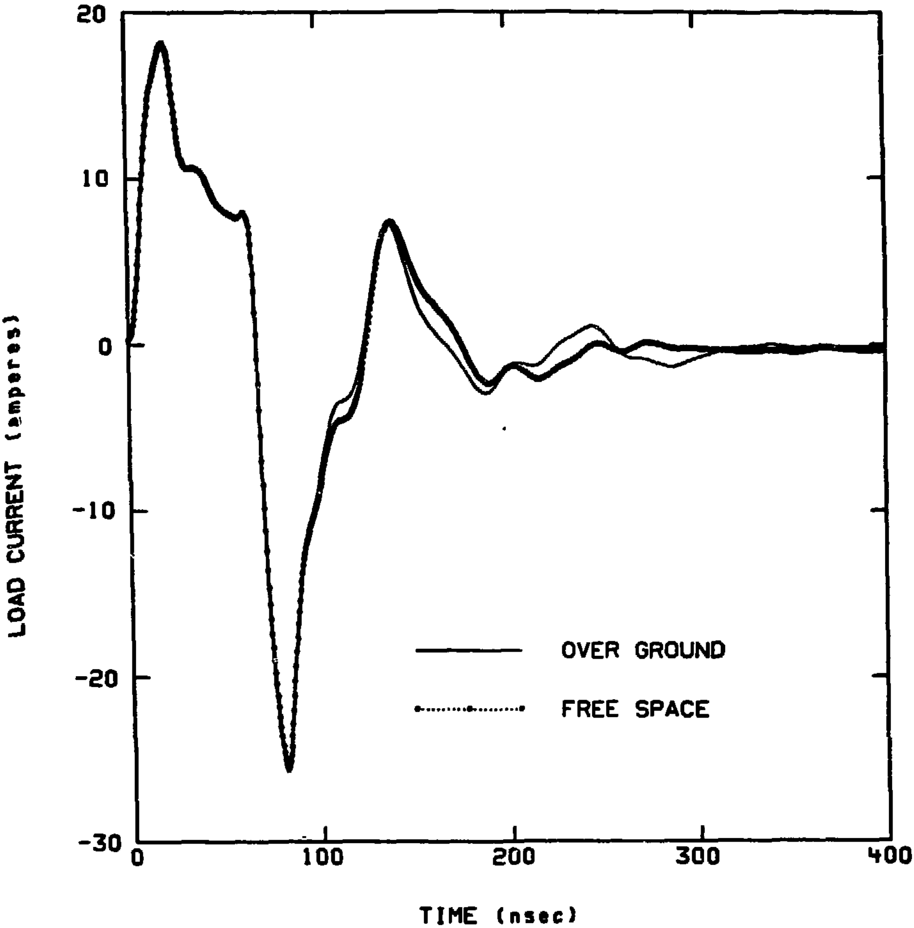
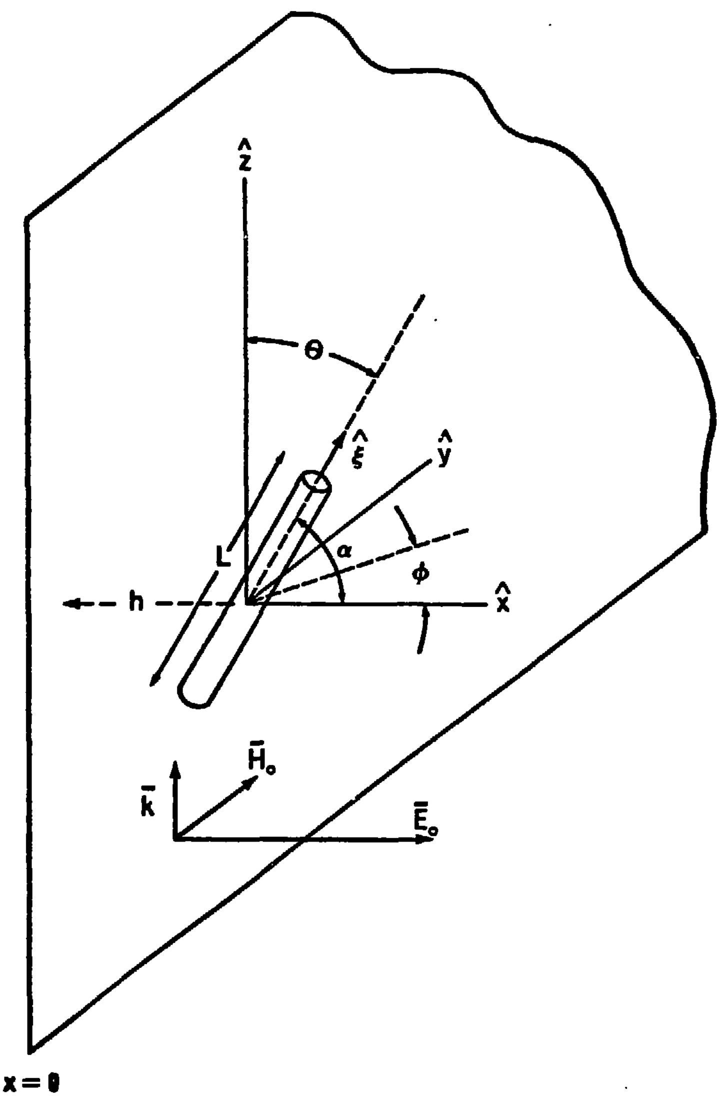


FIGURE 9f. Comparison of time responses (d) and (e) (after Deadrick, Landt, and Miller, 1973).



Scattering from a conducting wire located near a perfectly conducting ground plane:

FIGURE 10a. Problem geometry

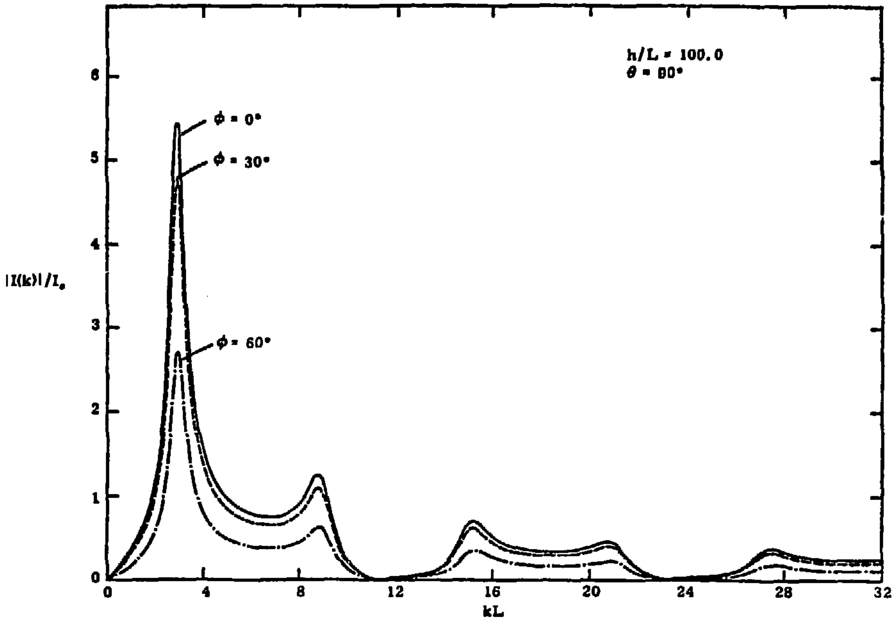


FIGURE 10b. Transfer admittance

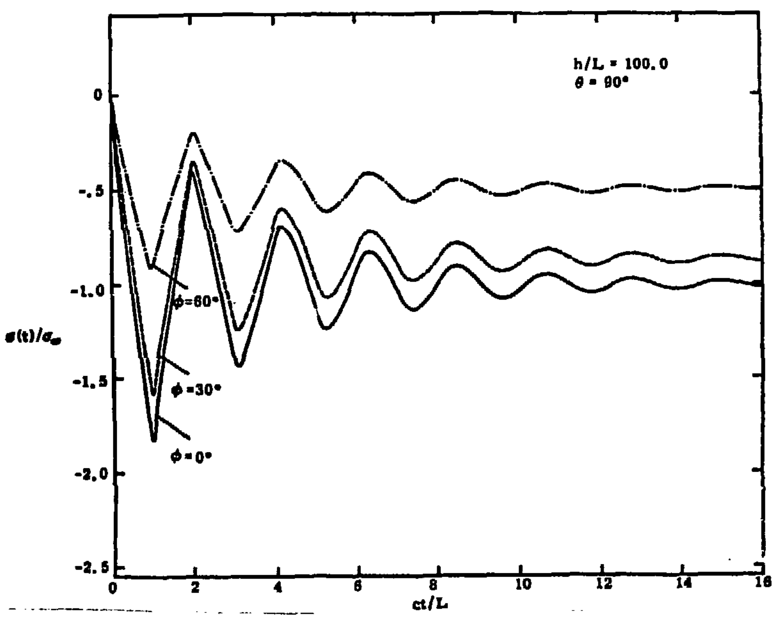


FIGURE 10c. Time response at $Z/L = 0.5$ for step wave excitation for $h/L = 100$

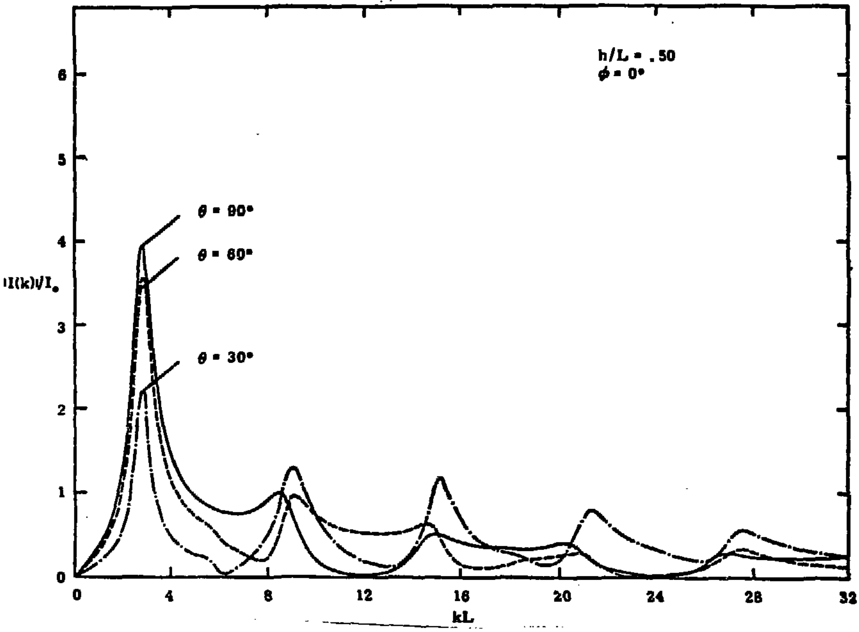


FIGURE 10d. Transfer admittance

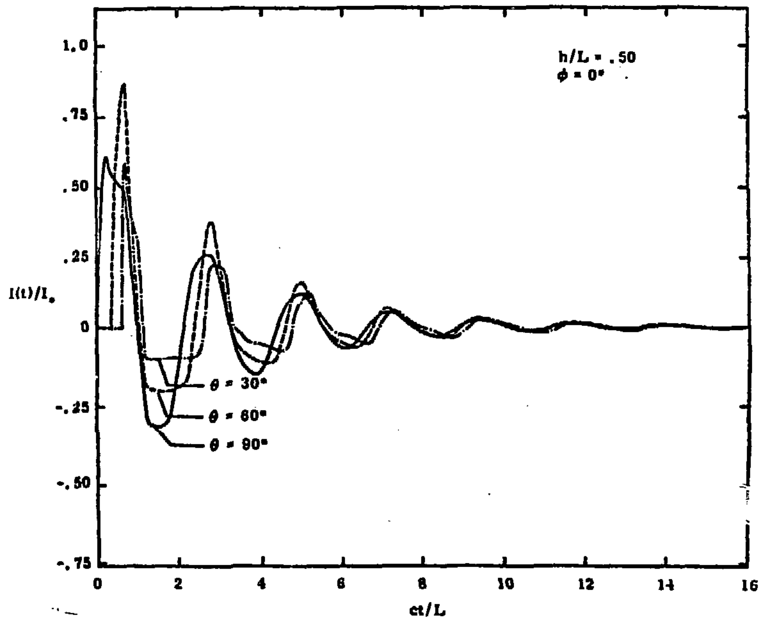
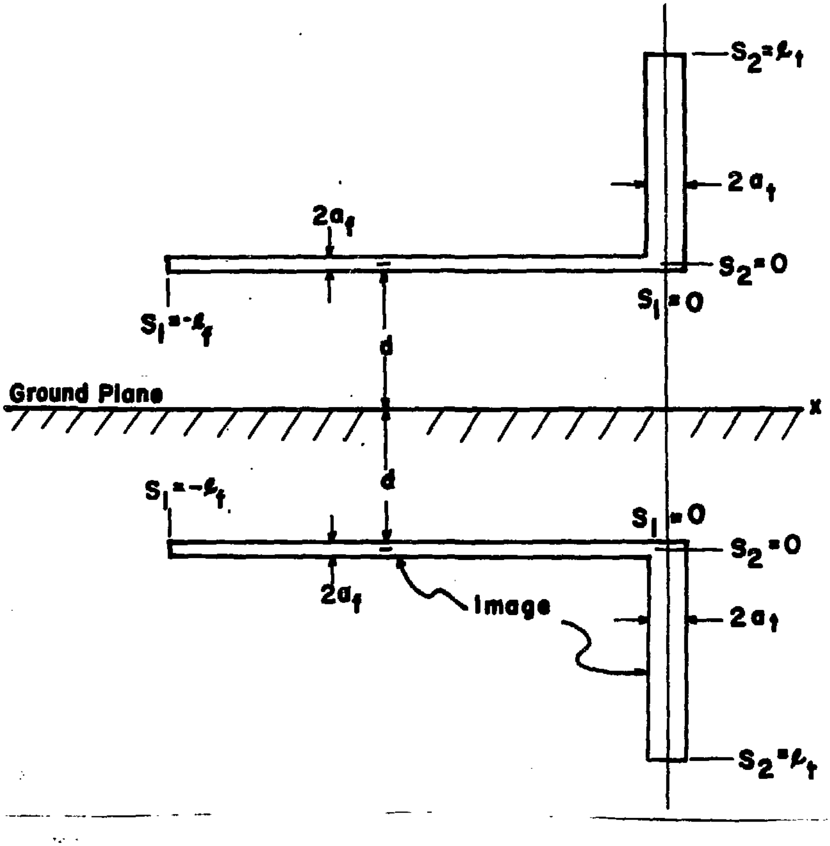


FIGURE 10e. Time response at Z/L = 0.5 for step wave excitation for h/L = 0.5 (after Tesche, 1972B)



Analysis of an L-shaped wire over a perfectly conducting ground plane:

FIGURE 11a. Problem geometry

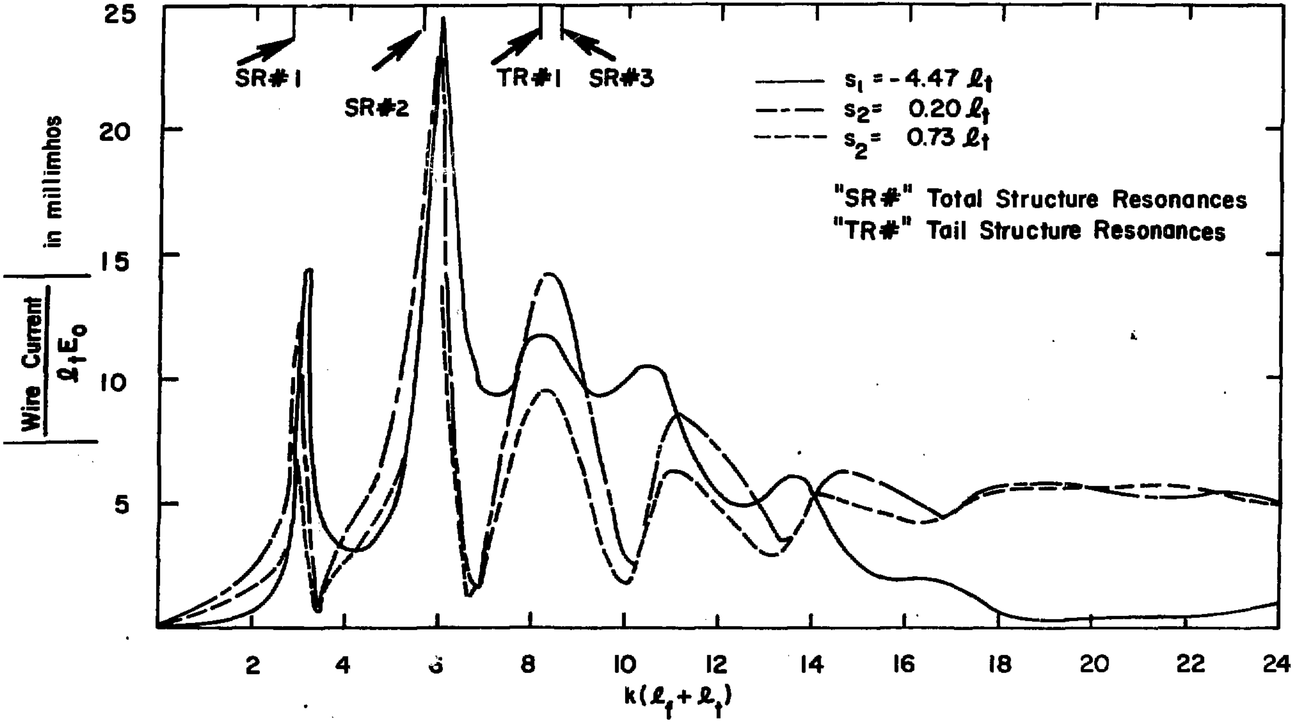


FIGURE 11b. Calculated frequency response at 3 points on the L-wire for $\ell_f/\ell_t = 5.133$, $a_t/\ell_t = 0.0781$, $a_f/\ell_f = 0.0279$, and $d/\ell_t = 0.375$ (after Taylor, 1972).

We conclude this series of graphs with some representative results due to Tesche (1971) for the parallel plate problem. The problem geometry is shown in Figure 12a. Two incident field types were considered by Tesche, center point source excitation for the antenna problem and plane-wave incidence for the scattering problem. Plots of the antenna feedpoint admittance for the former and of the center current (transfer admittance) for the latter are presented in Figures 12b and 12c as a function of the plate separation and with the wire angle relative to the plates a parameter. As expected the plate influence decreases with increasing plate separation and is generally speaking a minimum when the wire is perpendicular to the plates.

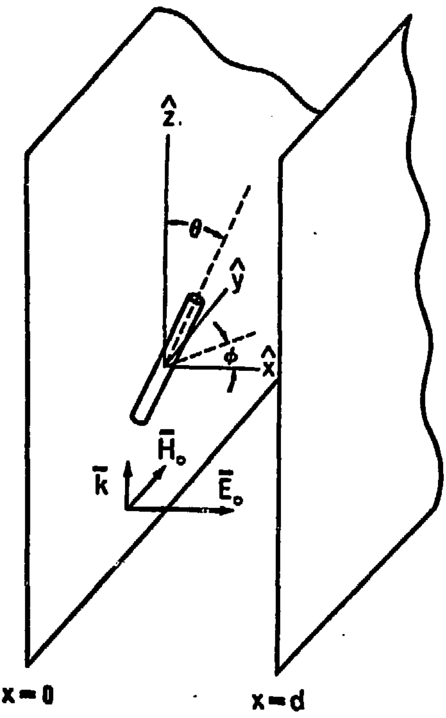
3. Aircraft Geometries

A still more complicated problem for analysis is the aircraft geometry, even in the lower part of the EMP spectrum, where the aircraft can be approximated by a thin wire model. Work has been performed in connection with the aircraft type problems at Boeing [(Curtis, et. al.) at North American Rockwell (Yang (1971))], and elsewhere adapting techniques previously developed for antenna and scatterer analysis. Efforts connected with the aircraft problem have essentially concentrated on low frequency representations in terms of thin wire structure with attempted extension to the higher frequencies using wire grid models. Results obtained in terms of the simple wire structures themselves appear reasonably accurate up to the frequency range of 10 to 20 megahertz, depending on the aircraft's size. A comprehensive review of all aspects of the aircraft assessment problem in connection with EMP may be found in the technology review report issued by Boeing (1972).

4. Surface Structures

As we have seen demonstrated above, wire integral equations find wide applicability to EMP-related problems. When however the wavelength-to-diameter ratio of the structure of concern exceeds ~ 0.2 or so, then a surface integral equation may be required. In this case, the integral equation must model the azimuthal variation of the longitudinal current and permit in the general case an azimuthal current flow as well. An approach using a surface integral equation for scattering from cylinders which are not small in diameter relative to wave length (a missile for example) are given by Kao (1970) and Harrison (1971).

Determining the surface current on a missile is but the first step in evaluating its EMP response. Generally speaking, what is required are the currents and voltages excited on transmission



Analysis of a straight wire located between parallel perfectly conducting plates

FIGURE 12a. Problem geometry

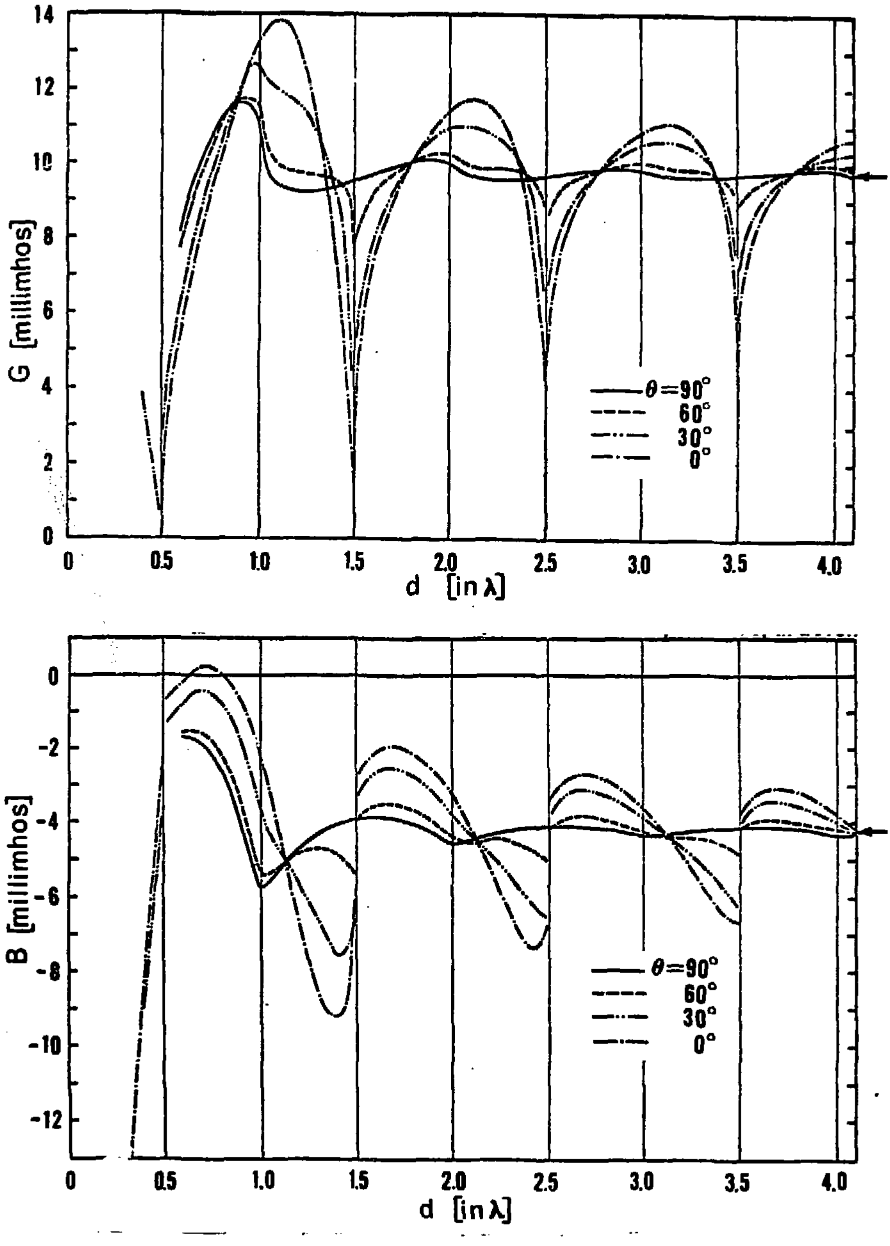


FIGURE 12b. Input admittance for L = λ/2 function of plate separation d

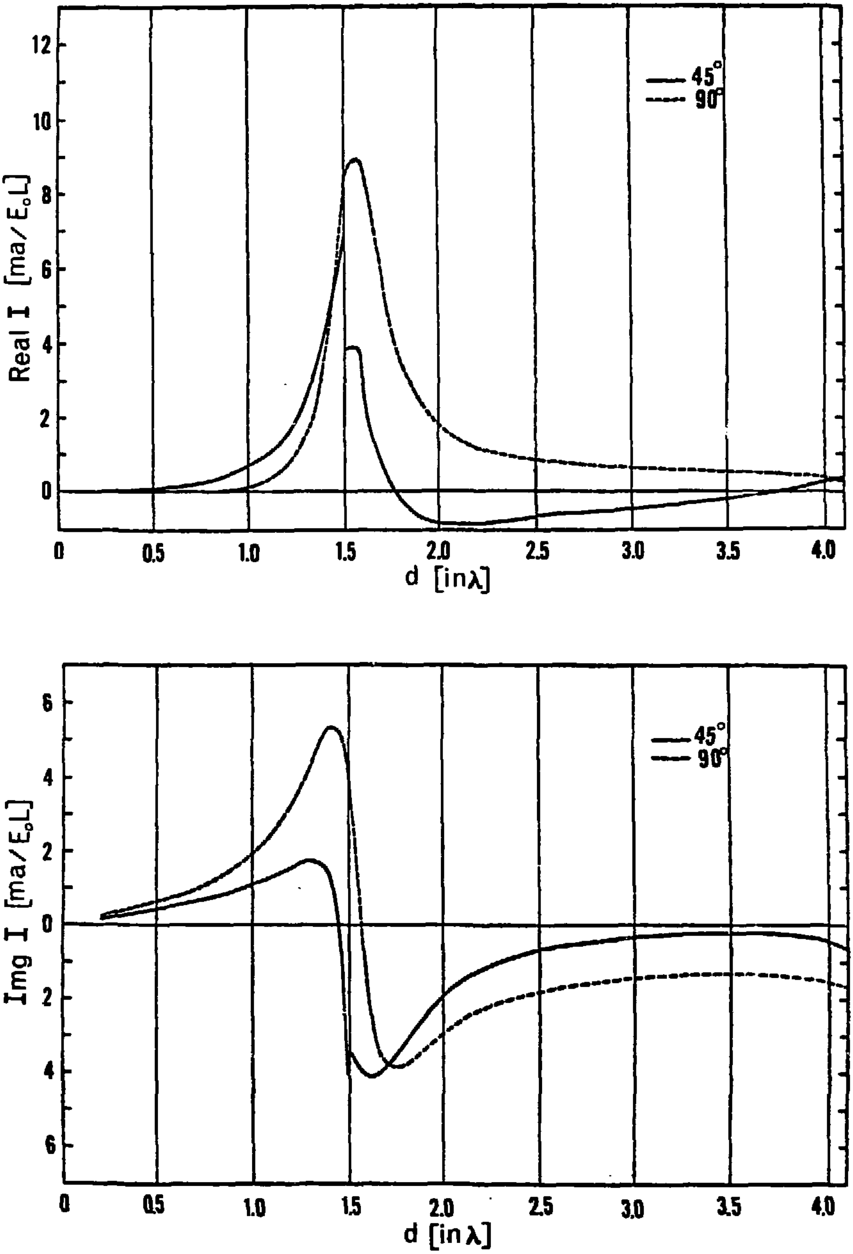


FIGURE 12c. Transfer admittance for TEM wave incidence and $L = 0.3d$, as a function of plate separation d, for $h = d/2$, $\Omega = 10$, $\phi = 0°$ and θ a parameter (after Tesche, 1971).

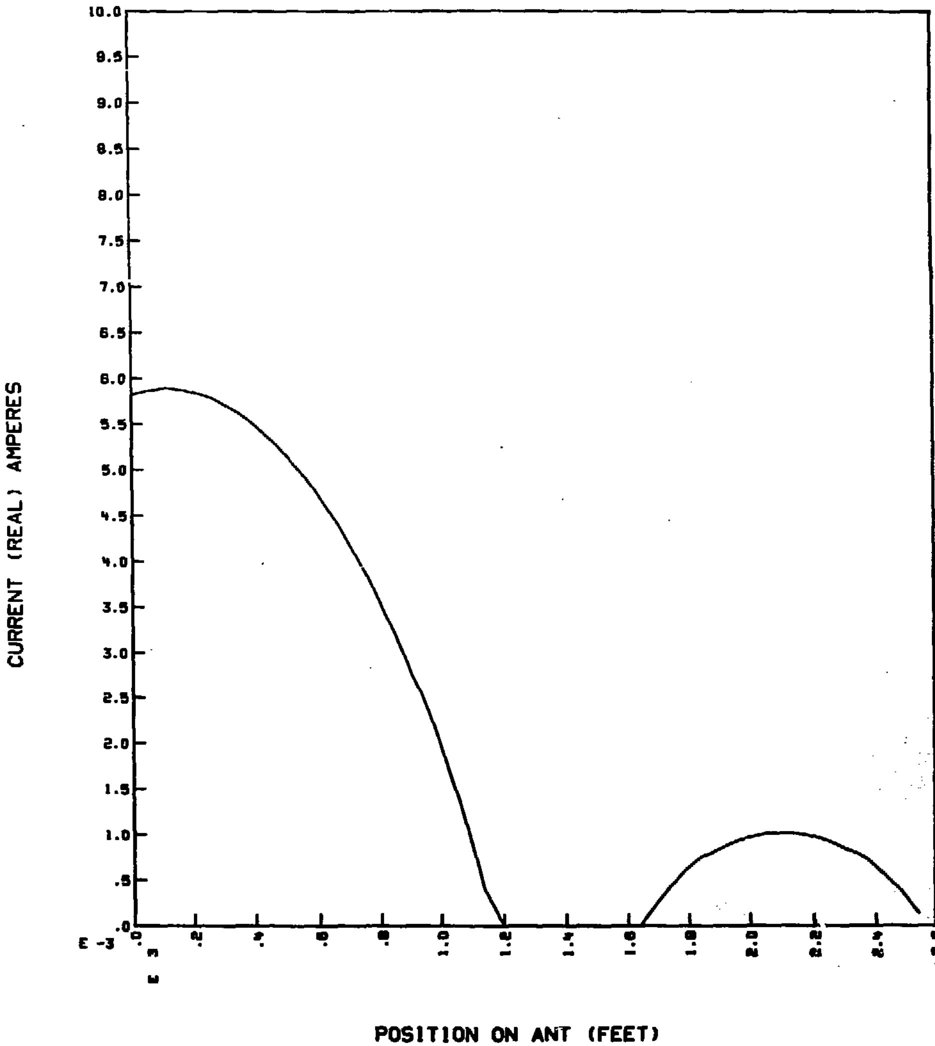
lines which, because of geometrical restrictions, may be located on the outside missile skin to provide the necessary interstage connection. Approaches to this problem have been presented by Harrison (1972) and King and Harrison (1972a, 1972b). In these studies approximate expressions are derived for the currents excited on transmission lines located in proximity to the rocket skin.

Various other surface integral equations have also been used for EMP problems. Some sample results obtained by Sancer and Varvatsis (1972) using the magnetic field integral equation for bodies of rotational symmetry are shown in Figure 13 for the case of plane wave scattering from a right circular cylinder. The problem geometry is given in Figure 13a, with the frequency dependence of the transfer admittance (for the total current) plotted in Figure 13b for incidence normal to the cylinder. The corresponding time response of the cylinder current for unit step excitation is presented in Figure 13c. Since this particular cylinder has a diameter-to-length ratio of 0.1, these results are not too dis-similar from those given previously for the wire case in Figure 2b. The principal difference is the more rounded nature of the wave form for the cylinder compared with the thin wire. Taylor (1971) has used a simplified version of an electrical field integral equation for a body of revolution to determine the total axial current induced on a truncated metal cone. Some results obtained by Taylor are shown in Figure 14. Part a of Figure 14 shows the cone geometry, with the axial current distribution given in Figure 14b for various angles of incidence relative to broadside.

5. Observations

These examples presented here for the most part deal with relatively simple geometries at relatively low frequencies. Clearly, solution of the general EMP problem for structures such as aircraft, buildings, ships, etc., is a long way off, at least in a rigorous sense. However, results such as those above permit insight to be gained and may point the way to defining engineering approximations that permit the efficient treatment of these much more demanding problems to the degree of accuracy required for their EMP survivability/vulnerability assessment.

In this connection it should be noted that the accuracy requirements of EMP problems are generally quite different from those with which we are used to dealing. The worse case approach, if it can be properly defined, would generally provide an acceptable means for answering the survivability/ vulnerability question. This in contrast to antenna impedance determination, where accuracy on the order of a few percent is generally sought, we may be perfectly willing to settle for



2-SEGMENT GAP INSULATOR MODEL (SEGS REMOVED) CURRENTS NORMALIZED TO 1-WATT

FIGURE 7f (Compare with Figure 7d)

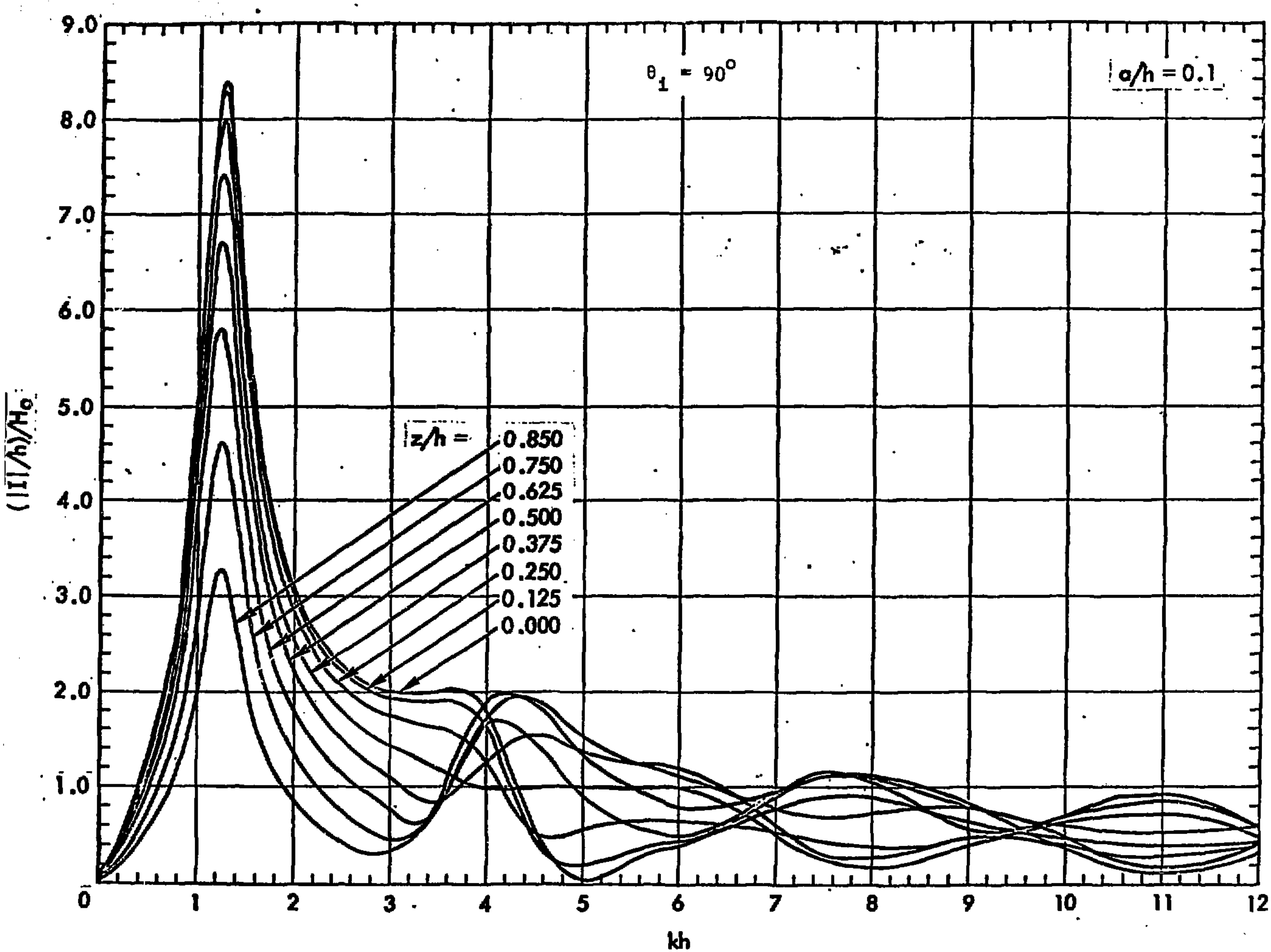


FIGURE 13b. Amplitude of the current density as a function of frequency at several positions on cylinder

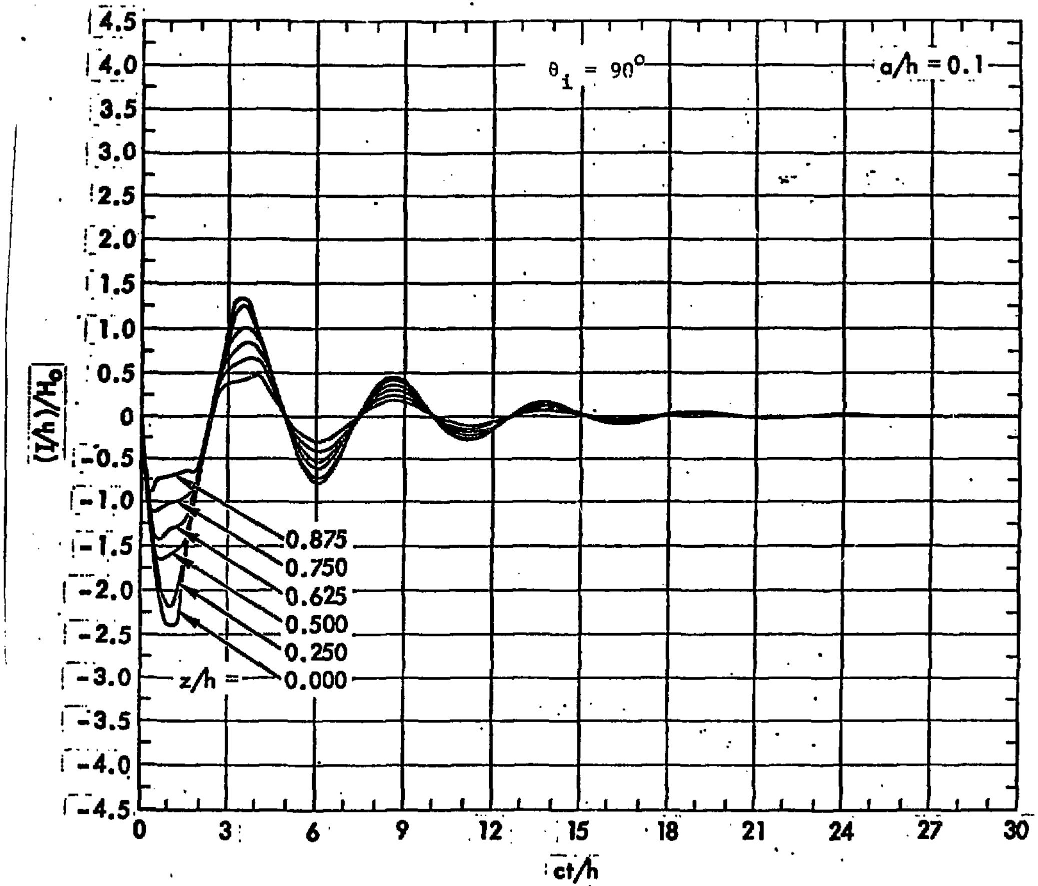
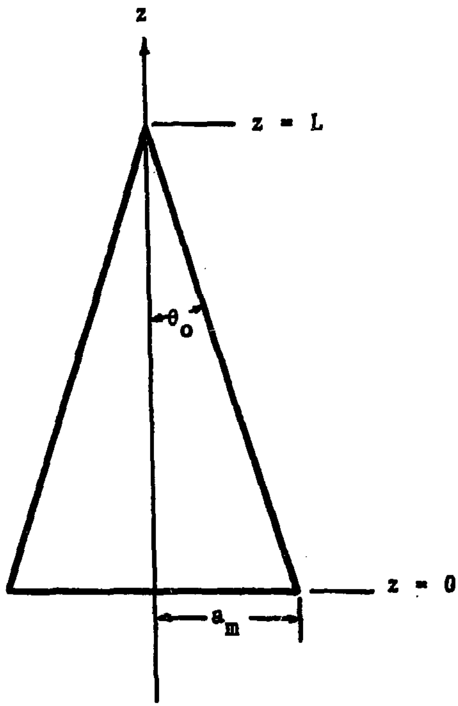


FIGURE 13c. The corresponding unit step time response of the currents shown in part (b) (after Saucer and Varvatsis (1972)).



Scattering from a flat-back metal cone

FIGURE 14a. Problem geometry

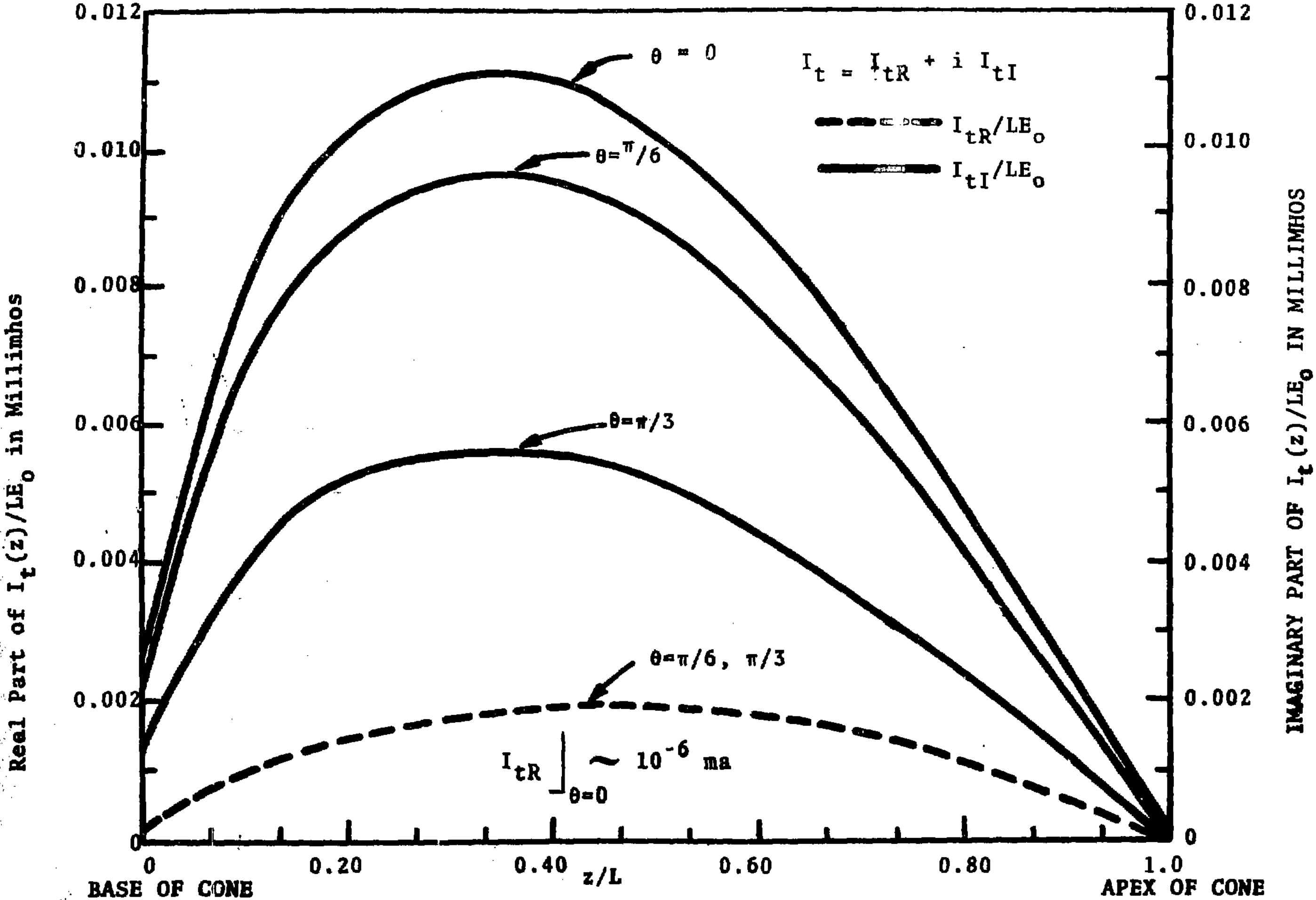


FIGURE 14b. Axial current distribution for various angles of incidence with $\theta_o = 5°$, and $kL = 0.1$ (after Taylor, 1971).

uncertainties of 3 dB, 10 dB or even less in the EMP problem. When the cost of analysis or testing to determine threat levels exceeds the cost of hardening to safety margins which allow for the inherent uncertainty in the actual threat, then further analysis may be clearly inappropriate. The EMP problem thus compels us to re-consider the EM analysis accuracy sought for a given problem in terms of an overall system requirement.

B. Exterior-to-Interior Coupling

The basic procedures outlined above may represent but the first step in one of many required to analyze the survivability/vulnerability of a particular problem. Ultimately required to do this is knowledge of the actual threats which may occur at the various susceptible components in the system. Obviously then, for equipment locations within the interior of the structure, such as is the case of avionics on an aircraft, or communications equipment in a communications center, it is necessary to determine how the energy which is received from the EMP wave form by the overall structure penetrates the interior.

There are three separate exterior-to-interior coupling modes which might be identified in this regard. They are:

1. Wall diffusion.

2. Aperture coupling.

3. Penetrating conductors.

Each of these coupling phenomena presents its own particular analysis problems to the EMP engineer.

The exterior-interior coupling aspect of EMP can, for only moderately complicated structures, increase the analytical scope of the problem to an extent far greater than that represented by the transfer admittance portion discussed above. We can offer little by way of example for the overall problem concerning this area since there has been relatively little concrete progress made to date. We can, however, offer some brief comments on the state-of-the-art in this area as it now exists.

We should furthermore observe that the transfer admittance approach which has been discussed is not a mandatory one in determining the threat at a particular point within a system interior. It may be possible, for example, to derive quantitative information concerning interior threats without explictely determining induced source distributions on the structure's exterior. The shielding effectiveness of various kinds of geometrical enclosures, for example the circular shell, can, of course, be found via modal expansions with

surface current and charge densities available if desired, but not actually necessary in determining interior fields. The transfer admittance viewpoint may be most appropriate when the structure which is illuminated serves basically as an antenna to funnel energy into definable transmission modes, such as connecting transmission lines which join an antenna and its receiving equipment.

1. Diffusion

Diffusion of the incident EMP fields through structure walls has been analyzed in various frequency regions for a variety of geometries using a variety of approaches. It has been found, generally speaking, that for frequencies such that the wave length greatly exceeds the maximum structure dimension, the interior fields are relatively insensitive to the actual structure shape and depend more on the structure walls thickness and maximum interior dimension. Furthermore, the magnetic field shielding effectiveness decreases to zero with decreasing frequency for most typical conductors, whereas the electric field shielding effectiveness becomes very high at low frequencies. Data concerning the shielding of circular shells may be found in Harrison and Pappas (1965). Their results for circular shells, which are not markedly different from the shielding characteristics of cylindrical shells or rectangular boxes, show that for frequencies exceeding 10,000 Hertz or so, the interior fields due to wall diffusion are generally less than 60 dB relative to the exterior amplitudes. When the structure walls consists of rebar and is thus not a continuous metallic surface, the shielding effectiveness may be considerably less and will, if the frequency becomes high enough, become largely ineffective since the rebar mesh openings are then large relative to the wave length. An analysis of rebar-like structures may make use of the wire mesh reflection and transmission coefficients derived by Astrakahn (1968), or by the more rigorous results presented by Dudley et. al (1972), where an integral equation procedure was used to determine the shielding effectiveness of an array of thin parallel wires.

2. Aperture Coupling

Coupling through apertures in the structure walls is vitally important for many typical problems. Again the B1 aircraft can be cited as an example, as can various of the missiles which have been studies, and of course the shielded structures used as communications facilities, even when provided with what are apparently continuous metallic walls. Wall openings are necessary for entry of required services to the interior. In addition, apertures may occur because of shielded doors being improperly seated, defects in welds, etc., all of which may provide points of entry via aperture coupling. Aperture

problems have of course been of interest for many years in connection with other EM problems as well, and a considerable degree of attention has been consequently devoted to these problems. Recent examples of aperture analysis oriented primarily to the EMP problem are due to Taylor (1973), and Harrison and King (1972).

3. Penetrating Conductors

The question of penetrating conductors is perhaps not as well understood as the prior two modes of exterior-to-interior coupling. As an example of a penetrating conductor we can mention a shielded power line whcih enters a shielded facility. Because of the finite shielding effectiveness of the various connectors, finite leakage impedance across the shield to the interior cable itself, etc., currents excited on the cable shield outside the structure may be injected onto the cables which continue through to the interior, and therefore provide a point of entry for energy to reach interior components. While a considerable amount of analysis has been devoted to the effects of various kinds of cable shields themselves, the particular problem of the conductor which penetrates the structrual wall has received relatively much less attention. At present it appears that experimentation via current injection measurement provides the only practical method for obtaining such information.

C. Interior-to-Interior Coupling

The preceeding two steps, transfer admittance determination and exterior-to-interior coupling analysis do not permit resolution of the threat determination at a particular interior system component by themselves alone. Since the interior geometry of most problems of interest is relatively complicated and may exceed the wave length over much of the frequency range involved, the question of energy redistribution within the structure must also be considered. This is a problem which has also received relatively little analytical attention because it is so extremely complicated. The analyst is faced with the very difficult problem of even defining or describing the geometry of the configurations in question, let alone performing meaningful numerical analysis as to the various current-voltage relationships which may be produced by the energy coupled to the interior. It is quite obvious at this point that what is required is a classification as to the relative importance of the various modes and points of entry to determine if, for example, a particular component is exposed to a higher energy input as a result of diffusion, aperture coupling, or penetrating conductors. Subsequent analysis may then permit dealing with only the dominant of these mechanisms with a consequent reduction in the complexity of the analysis, which in any case will still not be trivial.

While it may appear at this point that it is almost mandatory to consider approaching this problem from a worse case or statistical basis, if such a procedure can be defined, some success as mentioned above has been obtained in dealing with a particular problem from a fairly deterministic viewpoint by modeling the various cables and coupling paths involved, such as has been done on the MINUTEMAN Program. Again, this kind of approach takes advantage of the fact that one energy coupling mode is of predominant importance, in this case currents injected on cables, and thus relatively little attention is required for the other kinds of energy injection. Some preliminary analysis is of course necessary to determine which are the dominant forms of energy input mechanisms.

Some degree of appreciation for the extreme complexity of the real life EMP problem has hopefully been developed from the preceeding discussion. The transfer admittance calculation may be the simplest in terms of problem formulation and solution effort required. Evaluating energy coupling modes from exterior to interior present more complexity to the analyst, while interior to interior coupling characteristics may be most complicated. However, while the overall problem is so very difficult for rigorous analysis, it appears possible to achieve a degree of simplification which will permit one to effectively deal with its various parts while maintaining some degree of confidence in the results. Of course, testing must always play a role in providing the degree of confidence necessary to actually say whether a particular system is hard or not.

As another example in simplifying a complicated problem, we might mention some aspects of the B1 program. In this particular case, the first step in performing the analysis was to determine surface currents produced on the aircraft structure as a function of frequency. The exterior-interior coupling modes were then analyzed by viewing diffusion through the aircraft skin as of primary importance and assigning an attenuation value to the diffusion phenomena. For those avionic components contained within additional shields in the aircraft interior, a further attenuation value was assigned to characterize their exposures. Finally, then, a number for the current excited on a cable within the aircraft interior or within interior shields could be derived on the basis of the total structure current on the exterior, the shielding properties which were derived and a ringdown time. This particular approach has the merit that it leads to a well-defined threat and consequently, hardening criteria for the separate components. Note in this particular case no mention is made of energies associated with the interior field which are instead described entirely in terms of currents excited on cables. This seems reasonable in that it is eventually the currents which are excited on the cables which penetrate the various compartments and avionics that ultimately will damage susceptible components.

IV. CONCLUSIONS

Two things should have become clear from the preceeding discussion.

1. EMP problems are in general much more complicated than the usual radar scattering or antenna analysis with which the EM analyst is faced.

2. The concept of accuracy for EMP problem assessment is significantly different from that normally associated with scattering and antenna problems.

Fortunately these two points tend to offset each other in terms of rendering the EMP survivability/vulnerability problem assessment at least potentially manageable. While current numerical methods are not developed to the point of solving general EMP problems in any great detail or to any great degree of reliability, it does appear at present that these methods can be applied to various aspects of the overall problem, and do then, represent a tool useful to the EMP engineer. The further development of a methodology for the general treatment of EMP problems may find these numerical methods to be of greater use and, taken together with continued developments in the general area of numerical electromagnetics should lead to significant progress in dealing with the EMP problem.

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